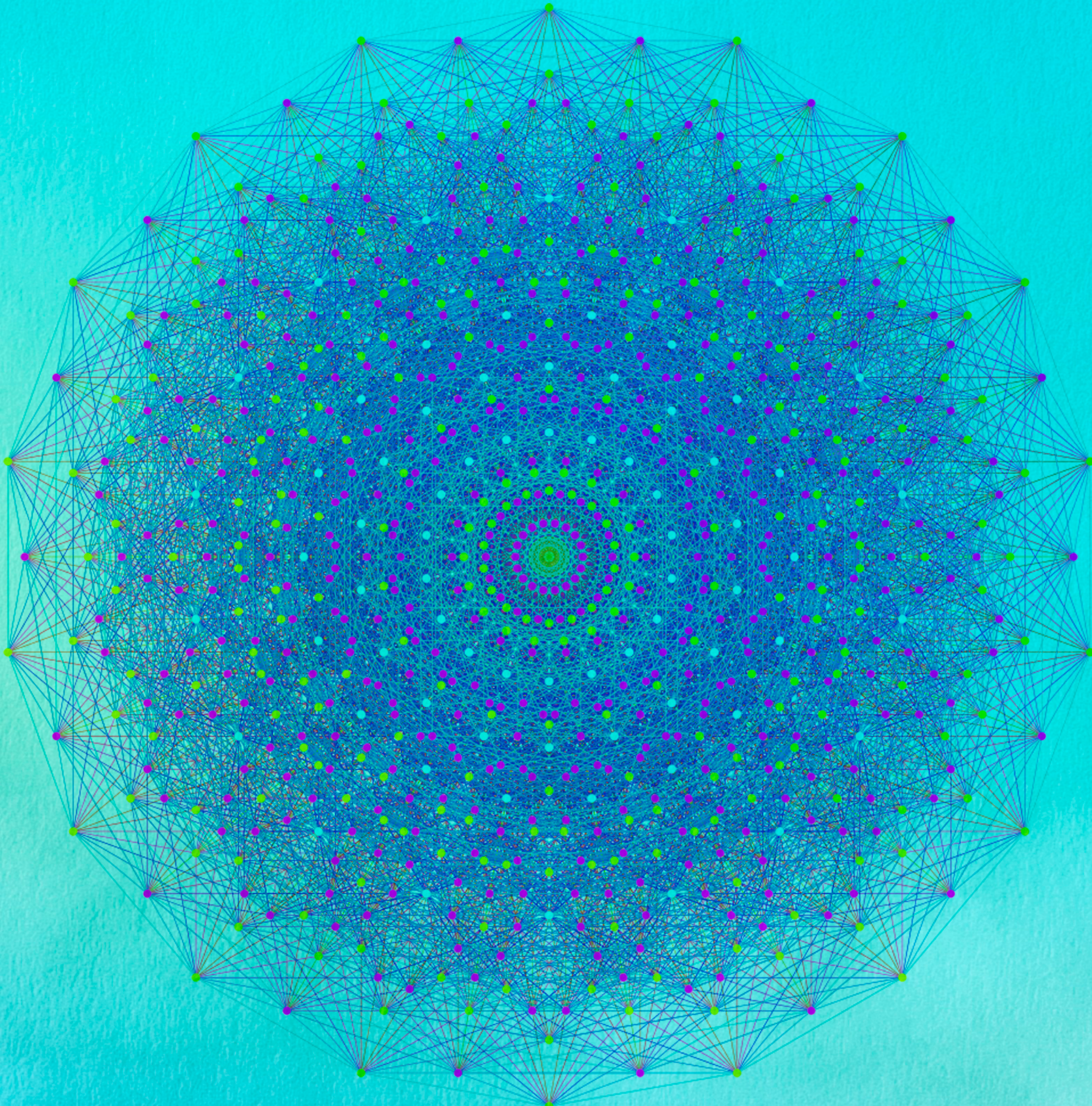
Proceedings of the Inaugural Conference of the Mathematics Teachers' Association - India

January 3-5, 2019



Homi Bhabha Centre for Science Education, TIFR

Mumbai, India

Preface

This volume contains the papers presented at **MTA-I-2019: the inaugural conference of the Mathematics Teachers' Association of India** held on January 3-5, 2019 at the Homi Bhabha Centre for Science Education in Mumbai.

The objective of the Association is to bring together teachers, educators and mathematicians in a joint endeavour, to sow the seeds for a long and sustained engagement with the complex task of improving mathematics education, fulfilling a long felt need for such an organization of Mathematics Teachers across the country. The Association aims to serve as an instrument to improve the standards of mathematics at various levels, generate interest in mathematics among students through various activities, nurture mathematical talent etc. Towards these objectives the Association plans to undertake various activities including development and adoption of learning materials and teaching strategies, publication of material related to mathematics teaching – including in particular a Newsletter, holding conferences, workshops, orientation programs etc, to name a few. MTA will also coordinate with other national and international bodies with similar objectives.

The inaugural conference provided a forum for mathematics teachers, mathematics educators and mathematicians from all over the country to come together and discuss all aspects of mathematics education, and also to offer directions for the newly formed Association. The conference saw discussions on mathematics teaching and learning at school level as well as college level. Discussions on mathematics education were on curricular as well as pedagogic aspects.

The conference honoured the 188th birthday of Savitribai Phule who pioneered Education for All in India, especially that of girls and all children from the poorest sections of society.

There were 84 submissions to the conference. Each submission was reviewed by at least 2 programme committee members. The committee decided to accept 38 papers, some for oral presentation, and some as posters (due to time constraints in the programme). The programme also included 3 theme sessions (Challenges in school mathematics, College mathematics and Classroom Experiences), 2 Workshops (Technology in Math education, Bridging school and college mathematics) and a Panel discussion on Engaging every student in Mathematics: realising the visionof Savitribai Phule.

I thank all members of the Programme Committee for doing an excellent job of putting together a rich set of presentations and discussions. Dr Jonaki Ghosh helped greatly in the Editorial work for the Proceedings, checking each submission carefully. All members of the Organizing Committee, and indeed the entire staff of the Homi Bhabha Centre for Science Education deserve praise and appreciation for the smooth running of the conference.

We thank the Indian Physics Teachers Association and the National Board for Higher Mathematics for generous support for the Association.

The Easychair system grealy helped ease the process of collecting, reviewing and putting together this volume, thank you Easychair.

We hope that this volume constitutes an imporant initiation of interactions between mathematicians, mathematics teachers from schools and colleges, educators and education researchers in India.

December 28, 2018
Chennai

R. Ramanujam
Institute of Mathematical Sciences,
Chennai

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Panel discussion

Engaging every child in mathematics: realising the vision of Savitribai Phule

Chair: Bhaba K. Sarma (IIT Guwahati)

Speakers:
Anita Rampal (CIE, Delhi University)
Kameshwar Rao (NCERT (Retd))
Dinesh Lahoti (Edugenie, Guwahati)

The dream of compulsory and entirely free school education for all children in independent India that was envisaged by the freedom movement and visionaries like Savitribai Phule has not been realised despite the enactment of the Right to Education Act. The change visualised for an enjoyable schooling for children by National Curricular Frameworks (NCF) has not translated into reality yet. Engaging every student in Mathematics is one of the biggest challenges of the universal school education in the country.

Needless to say, the state (central and state governments) has lot more to do for putting a robust education system in place meeting the challenges of quantity, quality and equality, where every child can have enjoyable learning experience and every teacher can perform to her best capabilities with dignity. Nevertheless, we, the mathematics teachers, too have a great responsibility in fulfilling the dream of quality universal education in the country.

In this session, we look forward for some fruitful discussion on curriculum and pedagogy of school mathematics and the best practices inside and outside the classroom to provide the child effective and enjoyable learning experiences.

The panelists ponder on issues including:

1. Reframing school mathematics that can actually engage all children in disparate and diverse socio-cultural contexts
2. Addressing upfront issues of social justice and inequality through the curriculum - following the spirit of Savitribai's work
3. Role of informal initiatives for math activities outside the classroom

CHALLENGES IN SCHOOL MATHEMATICS EDUCATION

-Athmaraman R

The whole essence of mathematics is abstraction. Hence it is natural to face challenges in making it understandable, usable and enjoyable—all at the same time, for all the learners in a math class!

Here is a thin outline of the challenges customarily experienced by teachers, students and the public.

Challenge 1

Mindsets: Many students have come to believe over time that they are just not good at math.

How are we going to change this psychological block?

Challenge 2

Prejudices hold sway against mathematics in general.

- "I'm not a math person."
- "Math is hard"
- "Math is for nerds"
- Math is a bundle of mere computational or arithmetic skills.
- Math is a package of rules to memorize.
- Math is foreign to everyday life

Challenge 3

Teaching the teachers, shallow content & **Pedagogical tight spots:**

Learning outcomes are no longer a checklist of skills that students need to "do" by the end of a grade level.

We want teachers to have students solve problems their own ways and have those students defended their processes. What if a student solves a problem in a way the teacher cannot follow?

Many teachers believe that the teaching of mathematics does not lend itself to a great deal of varied instruction.

Lack of awareness and usage of a range of contemporary ICT resources suitable for the classroom by teachers is damaging the learning process.

How do you make use of it? Do you trust in it? Do our teachers possess the skills in using ICT for communication, presentation, task preparation and implementation?

Challenge 4

Resources:

Are teachers trained to appreciate the positive impact on student learning afforded by making connections and applying mathematics to 'real-world' issues through problem-based investigations and integrated approaches?

One should not assume assume that just because a math problem has a "real-world" connection that the problem will hold "real interest" for learners. A problem of exponential growth to calculate retirement savings based on compound interest is very real-world to a teacher, but few teen-agers can relate it to retirement savings.

Gone are the days where teachers just followed the text book; they have to become seekers. If you seek, you will find.

Challenge 5

Cultural and Political dilemmas

How are we to address parents' concerns as math doesn't look the same as when they were taught?

Are the teachers being restricted to pre-made report card comments?

Teachers try to align their thinking and practice with State and school policies around assessment, particularly with regard to accountability

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You can go on adding to the list of challenges. To put in nutshell:

What math teaching is:	**What math transaction should look like:**
• Teacher talking and giving examples • Lots of practice of types of questions	• Teacher posing meaningful task, listening and responding to student thinking, connecting mathematical ideas, helping students to consolidate their understanding
• Little understanding as to why	• Developing both conceptual understanding and procedural fluency.

So there are challenges and hence problems. How are we to solve them?

We will be listening to three suggestive points of view in this session; all relate to school level mathematics.

In the first one by Dr Anna Neena George, a pedagogic perspective is given on 'lack of understanding' in math problem solving.

The second one by Sneha Titus is about developing divergent thinking among students for creative approaches in teaching and learning mathematics.

The final one by Dr Santhanam is on heuristics of Problem posing, essential for productive math learning.

The other aspects of this theme may be reserved for conferences and seminars to follow this inaugural one.

Abstract

THE LACK OF UNDERSTANDING LEADING TO POOR MATHEMATICAL PROBLEM SOLVING IN SCHOOL CHILDREN

Dr. ANNA NEENA GEORGE

ASSOCIATE PROFESSOR
GVM'S DR. DADA VAIDYA COLLEGE OF EDUCATION
PONDA-GOA

Mathematics is considered the most quintessential subjects and also the most formidable. It is included from kindergarten onwards with the intent of developing problem solving ability, logical reasoning and analytical thinking. However, the teaching and the learning of mathematics seems to be restricted to memorise the algorithm and aim for the 'right answer'. The understanding of the problem and the concepts have been relegated and replaced with extreme emphasis to speed of finding the 'right answer'. The very crux of mathematics teaching is to develop problem solving skills and to apply it in real life context.

Cobb et al. (1991) suggested, the purpose for engaging in problem solving is not just to solve specific problems, but to 'encourage the interiorization and reorganization of the involved schemes as a result of the activity' while Schoenfeld(1994) opines the conventional learning of mathematics only enables students to perform algorithmically and understand mathematics without reasoning, Jenning and Dunne (1999) have expressed the view that most students have difficulty in applying mathematics in real-world situations and Van den Heuvel-Panhuizen (1988) argues that students will most likely fail to remember the concepts and will be unable to apply mathematical concepts. These findings strongly point towards the purpose of teaching mathematics not being fulfilled.

In this paper, the discussion is on how students can be made to understand the problem, the algorithm and made more responsible for their own learning. They need to construct their own problems and even devise own strategies. The most missing part of mathematics teaching learning is the discussion of the problem which facilitates understanding of the problem. In this paper the lost crucial element of mathematics learning 'understanding the problem' is emphasized.

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SNEHA TITUS – Talk abstract

OPEN SESAME!

If one were to find the words *Problem* and *Solution* in a 'Match the Following' exercise, the easy way out would be to map the former on to the latter. But what if the plural *Solutions* was in column 2? Along with the word *Answer*. That would be a puzzler. Surely every mathematical *Problem* has an *Answe*r? The concept of there being no answer or multiple answers seems far-fetched and quite frankly, rather futile in the mathematics classroom. This is true of the classroom where mastering techniques of problem solving is the focus. Sadly, we have seen the limitations of such pedagogy, however effective it may be for harvesting marks. Ranks and marks are tangible 'proofs' of good teaching- unfortunately, the number of good thinkers who are completely deterred by their lack of success in finding the right answer are not counted. If they were, all of us would know that we have little to be satisfied about.

How can we turn this situation around? Of course, sweeping changes are being made at the administrative level. But what can we, as teachers do? It is often the small steps that reverse trends and one such small step is the technique of using problems that have multiple solutions – popularly known as 'open-ended problems'. These are an effective way of focusing on process rather than product. Not just that, it celebrates the slow thinker, the creative mavericks, the methodical investigators and a host of other types of students. Not just the first one off the block!

In this paper, I focus on what open ended questions are, the advantages of using them and how teachers can use them as an effective assessment tool. Given that most of the problems that we find in text-books are closed-ended, we finally look at how to open up such problems. We deal with a variety of problems across different levels of the school curriculum. And we also investigate the advantages of setting problems which have no solution!

CHALLENGES IN SCHOOL MATHEMATICS

(Theme Session: MTA Conference: 3th – 5th January 2019)

Dr. S.R.Santhanam (AMTI – Chennai)

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Abstract:

Problem Solving and problem posing are two important components of school a mathematics teacher. Most of the teachers are proficient in problem solving but only few are good at problem posing or coining a new problem. In this theme session, how to coin a problem in all levels (primary, middle school, high school and higher secondary) will be discussed. It is a common feature that in Mathematical Olympiad Examinations, the year number will appear in one of the problems. In this talk an attempt is made to expose to the teacher delegates the heuristics of problem posing.

Keywords: Problem solving, problem posing, school mathematics

Resume - DR. ANNA NEENA GEORGE

Dr. Anna Neena George,Associate Professor, at GVM's Dr.Dada Vaidya College of Education, Ponda Goa, for 25 years,since 1994. Born and brought up in Baroda, Gujarat,studied at Gujarat Refinery English Medium school and Maharaja Sayajirao University (MSU),Baroda. Awarded 2 gold medals for M.Ed. in 1993 and Ph.D in 2003 on the topic, **Mathematical Backwardness and its Remediation in Goa**, from MSU,Baroda.

Have been teaching Educational Psychology, Environmental Education, Methods of Teaching Science to the B.Ed. students, guided M.Phil, M.Ed. dissertations, Counsellor for IGNOU and NIOS. Recognized as, guide for research in Education by Goa University.
Completed projects for ICSE Board,e-content in biology for std. 10th in 2012, with Learning Links,Mumbai and prepared Science Activity books for class 1 to 4,under SSA scheme for LEP (Learning Extension Programme) in 2013.

Ongoing UGC sponsored Minor Research Project : To study the effectiveness of 5e Model of Teaching Mathematics on standards seven students and the challenges faced by the teachers in using 5e Model.

Additional Responsibilities at the college

- Coordinator of Science Popularising Project (SCIFUN) for middle and secondary schools of Goa, funded by Dept of Science and Technology ,Goa from 2011 onwards.
- Designer and coordinator of 1 year course : GVM's Pre-primary Teacher Education Course from 2005 to 2017.
- Designer of the syllabi for the Diploma and Advanced Diploma course, Preprimary Teacher Education Course under OA 21, of Goa University, Community College in 2016

Resource person within Goa

- Resource person for SCERT, Dept of Extension Dr.Dada Vaidya College of Education, various School Complexes,HISTAG,Navodaya Leadership Institute,IGNOU(B.Ed.)

Resource person outside Goa

- Resource person for Oxford University Press (Gujarat,Maharastra,M.P.,Goa), Sangamitra school Hyderabad, Podar International school, Sangli, Vijaya college of education, Banglore, St.Pious X degree and PG college for Women, Hyderabad.

Writes articles for Teacher Plus, Edutracks, educational and research journals. Teacher plus included 2 articles in their e-books.

George, A.N. (2012) Nurturing the spirit of inquiry. Teacher Plus. Secunderabad, January 2012, ISSN No. 0973-778

George, A.N.(2008) Math in everyday life. Teacher Plus. Secunderabad, June 2008, ISSN No. 0973-778

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SNEHA TITUS

Works as Asst. Professor in the University Resource Centre, School of Continuing Education, Azim Premji University.

Sharing the beauty, logic and relevance of mathematics is her passion.

She is the Associate Editor of the high school math resource ***At Right Angles*** and she also mentors mathematics teachers from rural and city schools.

She conducts workshops in which she focusses on skill development through problem solving as well as pedagogical strategies used in teaching mathematics.

She may be contacted on sneha.titus@apu.edu.in

Dr. S. R. Santhanam

☆ Educational Qualification

B. Sc., Physics

B. Sc., Mathematics

M. Sc., Mathematics

M. Ed.,

Ph.D in Mathematics Education.

☆ Experience

42 years.

Started the career as an Asst. Prof. of Mathematics.

Principal of a School for 11 years.

Former Secretary-AMTI.

Conducted around 30 workshops for teachers in various parts of India and Gulf countries.

Resource person for RMO, INMO for 25 years.

Theme : College Mathematics

Speakers:
Shobha Madan (IIT Goa)
Fozia Qazi (IUST, Jammu and Kashmir)
Amber Habib (Shiv Nadar University, Delhi).

Chair: **B. Sury (Indian Statistical Institute Bengaluru)**.

Abstract

The pros and cons of undergraduate mathematics curricula are discussed. Typically such discussions are confined as to the content that is to be conveyed in the classroom and how it would be beneficial to students in terms of career options. Occasionally, pedagogy may be addressed, but this is quite rare for university education as compared to school education. Our speakers raise a different set of issues, centered on a common concern for not just conveying content but supporting and inspiring students towards a life with mathematics. This involves activities outside the classroom or the curriculum, a fresh look at the curriculum itself, and also the willingness to place the student above the curriculum.

One of the speakers analyses ideas about what needs to be included as mathematics education outside the confines of classrooms, exams and grading, but still runs close to the regular teaching. There is also be a discussion on how to carry out this activity seamlessly.

Another speaker talks about what it is like teaching mathematics on the back foot. In particular, she discusses the key objectives she had in mind while starting an actuarial program at her university and the success of the program in light of the fact that she and her colleagues have to teach in a constant state of uncertainty, curfews and strikes where they play catch up with the content all the time.

The third speaker takes up examples of undergraduate curricula with a focus on the gap between what the planners hope for and what the actual impact on students is. He analyses what we can do to take care of individual students while pursuing a common, general plan.

Theme : Classroom Experiences

Speakers:
Sadiya Rehman (Springdales School, Delhi).
Vinayak Sholapurkar (S. P. College, Pune).
Kanchana Suryakumar (Poorna Learning Centre, Bengaluru).
B. Sury (Indian Statistical Institute Bengaluru).
Chair: **Shobha Madan (IIT Goa).**

Abstract

The objective of this session is to highlight classroom experiences of mathematics teachers in schools and colleges.

One of the speakers discusses how students who perform similarly in class, especially high achievers in Mathematics, actually differ a lot when it comes to their Mathematical Identities.

Another speaker teaches in an Alternative School. She talks about her experience of learning from detailed corrections of notebooks.

One speaker shares the lessons learnt from his encounters with some extremely talented students in a Research Institute and with College students during refresher courses or talks. He talks about teaching of *Problem Solving*, and the positive effects of this on students.

Another speaker talks of the opportunity he has had to interact with students on a wide variety of platforms: regular undergraduate and post graduate classes in his college, the MTTS programme, Olympiad programme, Foundation programme (for school students) at Bhaskaracharya Pratishthana, and so on. These experiences have helped him grow as a teacher, offering him moments of unadulterated pleasure. Comparing his undergraduate classes in college and MTTS classes helps him analyse teaching learning processes, student responses, their progress and self-learning. The talk illustrates classroom discussions, question - answering, inter-student dialogues. They may be highly rewarding or chaotic ! A brief mention of experiences in Mathematics Olympiad classes is interesting as well.

Workshop on Bridging the Gap Between School and College Mathematics

Speakers:
Shobha Bagai (Cluster Innovation Centre, Delhi)
Anisa Chorwadwala (IISER, Pune)
S Kumaresan (Univ of Hyderabad (Retd))
Revathy Parameswaran (PS Senior Secondary School, Chennai)
Bhaba Kumar Sarma (IIT, Guwahati)

Chair: **Geetha Venkataraman (Ambedkar University Delhi)**

There are multiple facets and complexities when we delve into the topic of the workshop. These are explored through two lectures and a panel discussion. The lectures by Professors Kumaresan and Shobha Bagai take up topics in Mathematics that students find difficult to handle when they move from school to college. Panelists Dr Anisa Chorwadwala, Dr Revathy Parameswaran and Prof. Bhaba Sarma shed light on the many aspects related to bridging the gap from their experiences in School, Undergraduate and Postgraduate mathematics teaching experiences.

In his lecture, Professor Kumaresan discusses the gaps between high school mathematics and college level mathematics and the problems faced by even the good and motivated students. He suggests some remedial measures based on his experience in MTTS camps.

Professor Shobha Bagai's lecture is titled *Metamorphosis of Senior Secondary School Mathematics to College Mathematics: Special emphasis on Linear Algebra*. Transition from senior secondary school to college is an exigent demand on students, both personally and academically. In the context of mathematics, at the school level, a student is exposed to mainly numerical, formula based, intuitive argument aspect of mathematics. Once they enter the college system, they are lost in the abstractness of mathematics. Linear Algebra is one such course which is introduced in the inceptive stages of the undergraduate curriculum. The talk focuses on the pedagogy of teaching some basic concepts of Linear Algebra with particular emphasis on tasks that are productive for learning. These tasks may be adopted either at the undergraduate level or at the senior school level where matrices are introduced.

Panel discussion:

The panelists Dr Anisa Chorwadwala, Dr Revathy Parameshwaran and Professor Bhaba Sharma cover aspects related to XII grade curriculum paving the way for college education, the difference in nature of Mathematics up to Class XII and that of BSc onwards. There is some focus on expectation versus reality, and a discussion about courses on foundations in mathematics which bridges the gap between the students' preparation of school mathematics and the expectation on her maturity to understand the mathematics taught at undergraduate level. All panelists draw upon their experiences as teachers as well as during their student days.

Workshop Session 1: Technology in Mathematics Education

Session Chair: Dr. Jonaki B Ghosh, Lady Shri Ram College for Women, University of Delhi
Workshop resource persons: Ms. Sangeeta Gulati, Sanskriti School, New Delhi
Dr. Aaloka Kanhere, Homi Bhabha Centre for Science Education, TIFR, Mumbai
Dr. Ajit Kumar, Institute of Chemical Technology, Mumbai

Abstract

One of the most fundamental impacts of technology over the last few decades has been its contribution in pushing the boundaries of knowledge in almost every field. Considering its omnipresence it is no wonder that technology should form an integral part of education in our society. We may argue that if properly used, technology can significantly impact teaching and learning of most subjects, especially Mathematics. This brings us to some very important questions which educators and education systems world over have been faced with – what should technology be used for as far as teaching and learning is concerned? How can integration of technology be made most effective? These questions pertain to education in general and finding answers to them are certainly not straightforward. A very limiting view of the use of technology is to do the same things that we are doing today with greater speed and to streamline and replace routine procedures. However in order to reap the benefits of the full potential of technology we must use it to develop students' thinking and to introduce them to processes which cannot be done in the absence of technology.

Many countries across the world, in the last few decades have witnessed a major shift of paradigm as far as mathematics teaching and learning is concerned. Serious experimentation and research related to the integration of technology in mathematics education has in many ways revolutionized the way students experience the learning of mathematics today. Several research studies (Heid, 1988, 2001; Kutzler,1999; Lagrange, 1999) indicate that the appropriate use of technology can transform the mathematics classroom into a dynamic interactive learning environment where the learner can develop a deeper understanding of the subject and the ability to think mathematically rather than focus on practicing manipulative skills.

With the advent of new and powerful technological tools in the form of dynamic geometry software (DGS), computer algebra systems (CAS), spreadsheets and graphic calculators, students may focus on the processes of exploring, conjecturing, reasoning and problem solving and not be weighed down by rote memorization of procedures, computational algorithms, paper-pencil-drills and symbol manipulation. Some research studies (Arnold, 2004; Harwardeen, 2002) have shown that the appropriate use of technology can lead to better conceptual understanding and make higher-level processes accessible to students without compromising on paper – pencil skills.

Unfortunately in the traditional classroom, the emphasis is largely on developing procedural knowledge through drill and practice and on learning 'tricks' to solve problems. Getting exact answers is often over emphasized and the National Curriculum

Framework (NCF, 2005) in its position paper on *Teaching of Mathematics* recommends that we need to 'liberate school mathematics from the tyranny of the one right answer, found by applying the one algorithm taught.' The position paper also recommends that there should be a shift in the curriculum from 'content to processes'. It states that "content areas of mathematics addressed in our schools do offer a solid foundation.....What can be leveled as major criticism against our extant curriculum and pedagogy is its failure with regard to mathematical processes.....formal problem solving, use of heuristics, estimation and approximation, optimisation, use of patterns, visualisation, representation, reasoning and proof, making connections, mathematical communication.......". In order to achieve this shift, the position paper also advocates the use of technology in the form of computer software and calculators, "Technology can greatly aid the process of mathematical explorations,....If ordinary calculators can offer such possibilities, the potential of graphing calculators and computers for mathematical exploration is far higher".

Technology, if used properly, can be a great enabler in realizing the recommendations of the NCF. In fact technology can aid in the visualization of concepts, in exploration and discovery, in bringing the experimental approach into mathematics, in focusing on applications, in redefining the teacher's role, in helping sustain students' interest, in individualized grading and assessment, and in teacher outreach. But while technology has profound implications for mathematics education, it does not by itself supply solutions. The mere provision of technology in a class does not solve the problems faced in teaching. It is thus essential that serious research and experimentation go into the use of technology as an aid to teaching mathematics. In a world where technology is constantly evolving, it is imperative that mathematics teachers and educators develop an in depth understanding of the potential as well as challenges of integrating various technology tools in mathematics teaching and learning. Technology in the form of Dynamic Geometry software (DGS), Computer Algebra Systems (CAS), Spreadsheets as well hand held calculators and other devices are gaining some popularity at the school and college level. However, the large-scale implementation of such tools is fraught with many challenges.

This session of the conference addresses the benefits as well as challenges of integrating technology into the teaching and learning of mathematics. It brings up questions such as - How can technology be used to augment mathematical learning? What kinds of technologies are best suited for our mathematics classrooms? Can technology make mathematics accessible to all levels of learners? How can technology be used to make connections across topics? etc. Hands – on workshops on GeoGebra and MS Excel/LibreOffice Calc conducted as a part of this session familiarize participants with the pedagogical opportunities offered by these tools.

Highlights of the session

Use of technology in developing mathematical thinking: An overview

The session begins with an overview of different computer software available for mathematics instruction. Some examples of studies conducted in the mathematics classroom highlight the benefits and challenges of integrating technology. In some of these studies (Ghosh, 2015) students of secondary school used different digital tools to engage in processes such as exploring concepts, making and testing conjectures, looking for patterns, finding counter examples and dealing with multiple representations. Some studies also highlight active engagement of students in modeling activities and exploration of mathematical applications in which technology played a pivotal role.

Teaching and Learning Mathematics with GeoGebra - Hands on Workshop

GeoGebra is an interactive geometry, algebra, statistics and calculus application, designed for teaching and learning mathematics from primary school to university level. Tasks in GeoGebra may be designed to enable students to make conjectures and understand mathematical concepts. Multiple representations allow the learner to visualize and explore concepts, which are otherwise difficult and abstract. Translated by teams of volunteers, GeoGebra is available in over 60 languages (including Hindi and Tamil). It is a multi-platform application, available either through a web browser or as a stand-alone downloadable application that works across most devices. There are six perspectives of GeoGebra: Algebra, Geometry, Spreadsheet, CAS, 3D Graphics and Probability.

The first workshop of this session enables participants to experience the power of GeoGebra by creating and exploring applets in Geometry, Algebra and 3D Graphics perspectives. As a teacher-user one can use a pre-prepared applet to start a discussion in class. Drag vertices of a triangle constructed using polygon tool and show the measures of angles and their sum; what is changing and what remains constant? Is Pythagoras theorem true if regular polygons are drawn on sides of a right-angled triangle? Such questions will be answered using pre-prepared applets followed by the participants creating the same. As a student-user, step-by-step construction can lead to various explorations. The triangle centers are explored using this approach. It demonstrates that a dynamic worksheet created by the teacher coupled with thought provoking questions can prompt students to look for answers using the dynamic construction. Abstract concepts such as that of the derivative of a function as the slope of tangent at any point on the graph, are explored using trace of a point feature. The 3D perspective of GeoGebra which allows for visualization of 3D concepts are also highlighted. Finally, a short tour of the website www.geogebra.org familiarizes participants with enriching resources which may be used directly or may be customized.

Exploring Mathematics using Spreadsheets: A hands – on workshop

Spreadsheets such as Microsoft Excel or LibreOffice Calc can be very useful tools for performing scientific calculations and statistical data analysis. They enable us to simulate problems and also create, visualise and analyse interactive mathematical models. The second hands-on workshop of this session focuses on the use of a spreadsheet to solve, simulate and visualize some mathematical concepts. Problems discussed in this session include interesting paradoxes related to probability and randomness such as the Birthday Paradox and the Monty Hall problem. Graphical exploration of the Binomial and Normal distributions, simple regression analysis and optimization are also discussed.

The session concludes with an interactive discussion where participants get the opportunity to share their views on the use of technology in mathematics education.

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Storytelling as a pedagogic resource in the teaching of mathematics

-Models and illustrations

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Introduction

Stories have been termed as the 'primal act of the mind' (Wells, 1986). Schiro (2004) contends that our brains are story-seeking and story-creating instruments. According to Bruner (1996) human cognition is story-based and Egan (1986) explains that stories are majorly about affective matters. This implies that stories have the power to bring the intellect and the affect together. Owing to this power, stories have been used in the teaching of language for many decades. But, stories have been introduced in the mathematics classrooms only recently. The available research on the using of stories for teaching of mathematics can be grouped under five categories: a) Providing a contextual base for discussing mathematics(Anderson and Anderson 1995), b) Giving problem solving and problem posing opportunities(Zazkis and Liljedahl, 2009), c) Reaching all students including those with specific disabilities(Courtade et al. 2013), d) Providing motivational, attitudinal and emotional effects (Hong,1996); and e) Improving mathematical communication (Jennings et al.,1992). Thus, the existing literature supports the use of storytelling as a pedagogic resource and this paper discusses two models of storytelling, the Springboard model and the Epic story model (Schiro, 2004), that can be used for teaching mathematics. It presents examples of respective stories for the two models and mathematical content that can be interwoven into the story situations.

For the purpose of this paper, a story is being defined in the following terms. Each story must have a beginning where we introduce the audience to a problem. In the middle we complicate the problem and by the end the problem is solved by the story's characters.

The Springboard model

In the Springboard model, the characters and their situations in the story are used for beginning a discussion and after that the story is abandoned and mathematics is discussed independent of the story. The story in such a case is short, is narrated during a session and is used as a 'springboard', that is, an initiator for discussion.

To cite an example of a story for the Springboard model, I shall discuss the book 'Sir Cumference and the Dragon of Pi' (Neuschwander, 1999). It is a short adventurous story that introduced children to the constant ratio between circumference and diameter of a circle. The plot of the story revolved around a child named Radius whose father, Sir Cumference, had accidently turned into a fire-spitting dragon. Now, Radius had to figure out in one day's time the right dosage of a magic potion that will turn his father back into a person. The dosage puzzle was written on the potion's bottle as follows, *"The Circle's measure: Measure the middle and circle around, Divide so a number can be found, Every Circle great and small, The number is the same for all. It is also the dosage, so be clever. Or a dragon he will stay...forever"*

In the classroom, this puzzle could be given to the students of Grade VI to suggest ways through which Radius could find out the correct dosage to save his father. In the story students are introduced to the constant ratio of Pi, and Radius' quest comes to a happy end. Children can then proceed to other related ideas of finding circumference, diameter and radius of a circle. According to the Springboard model children abandon the story after one or two sessions and discuss the related mathematics independent of it.

The Epic story model

The stories in the Epic story model are interwoven with the mathematics and extend to around six to eight sessions. The characters in the story encounter multiple adventures, each building upon the previous ones, in terms of the story plot and the mathematics content.

The example of a story for the Epic story model that I am presenting in this paper, is based on a popular children's novel 'Alice in Wonderland' (Carroll, 1971). The story was modified to embed algebraic concepts in Grade VI so that the story of 'Ally in Algebraland' and learning algebraic concepts proceeded together. In the following paragraph, I shall briefly describe the story progression and the introduction of three algebraic concepts (finding unknowns, patterns and general rules; using a variable; notion of equal to sign) that were introduced to the students progressively during the story.

The story begins when Ally stumbles into a rabbit hole while following a curious looking white rabbit in a waistcoat who was rushing away for an important errand. To follow him further she had to seek entry into the Algebraland for which she had to solve many number puzzles, find matchstick patterns and rules between sequences of numbers. This was meant to be an introduction to algebra. An example of one of the number puzzles was as follows, *"Tell me who I am, I shall give a pretty clue, you will get me back...if you take me out of twenty-two"*. Ally could solve the number puzzles with the help of the students. Then she had to find and create matchstick patterns during this session and she could pass on these activities to the students. At this stage students can be asked to predict the number of matchsticks that would be required in the subsequent steps. Last step before gaining entry into Algebraland, Ally met a pair of twins, TweedleDee and TweedleDum. They told Ally that she had to earn points in the IN-OUT machine to go inside Algebraland. The 'machine' could be a worksheet shown below where students had to find the rules between two sets of numbers and guess the number in the third blank.

IN	OUT
A bird has 4 feathers	Now the bird has 16 feathers
A bird has 3 feathers	Now the bird has 9 feathers
A bird has ...... feathers	Now the bird has 64 feathers

The story proceeded to the next session, where Ally met the white rabbit. The rabbit mistook her for a financial auditor whom he was expecting. He handed her a small booklet containing many tables that had to be filled up by the auditor. The tables were meant to help the rabbit calculate cost of items in his departmental store because even though he knew the cost of each item, he could not find it for any number of items. Due to this he was making huge losses.

Number of books	1	2	3	5	m	K
Cost of books	Rs. 5	Rs. 10	Rs.	Rs.		

The example above is a table-building exercise for introducing students to one of the uses of a variable, that is, a generalised number. Students could be given many such exercises in class. The story proceeded after filling up and discussing the tables and Ally now wanted to handover the booklet to the white rabbit. When she looked up, he was gone already. He left an apology note but Ally was alone in Algebraland again. While she was contemplating her next move, she noticed some singing in the distance. Curious as she was, Ally decided to follow it. The sound was coming from a thatched hut in the woods. Ally introduced herself to March Hare and the Mad Hatter who were enjoying a tea party. When Ally inquired about the white rabbit, she was offered help on the condition that she would play the game of balancing the dangling table with them first. The dangling table was used to introduce students to the idea of an equal to sign where both right and left sides have to be balanced (equalised). Many more examples of balancing the two sides of the table can help students in understanding the idea of the equal-to sign. The teacher slowly introduced the writing of an equation and how each equation has a left and right side that need to be equal for an equation to be true.

In the next adventure Ally befriended some people who were finding solutions to (linear) equations (in one variable) and hanging their solutions on trees in order to please the Queen of Algebraland. The Queen found an error in one of the solutions given by Ally's friends and ordered their execution. When Ally protested at this unfair treatment she was challenged by the Queen to a Game of Equations. Ally accepted the challenge because the life of her new friends was in danger. The class was now divided into two teams, Ally's team and Queen's team. Each team was given one equation to solve and show how they got the answer on the board. That was how they earned points and the Game of Equations continued. After 5 rounds, the names of teams were interchanged. This way all students got to be on Ally's side. Even though Ally won the game but the story of Ally came to an abrupt end when she woke up from her dream. While she was asleep she had crossed many hurdles of algebra-based problems with the help of Grade VI students.

Conclusion

The Springboard model has a single problem at its core and takes very little time for narration and discussion, whereas the Epic story model has a long plot and 6-8 problems that are unveiled slowly as the story progresses. Thus, in the Epic story model students are curious about what will happen next in the story and remain intellectually and emotionally engaged with the story and the mathematics over the sessions. A teacher can use either of the models of storytelling to engage students with mathematics depending on the requirements of her students and the mathematical content.

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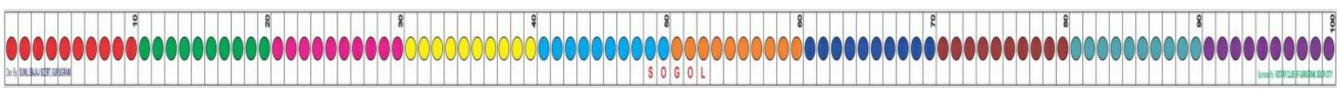
SOGOL – An Experiential Learning

Introduction

Children are not machines. They think but machines do not think. Children have interest and machine do not have . They work only as programmed. We should provide interesting and challenging opportunity so that they can learn. Sogol is one of interesting TLM.

In the NCF -2005, it is emphasized to follow child centered approach with a view that every child can learn Mathematics. But we know that Children learn only when they are interested. Creating interest is a big challenge for teachers. In NCF-2005, it is also expected that teachers should act as facilitators for learning by doing . They should think and give interesting opportunities with appropriate challenge to all children in which they learn through their own experiences. They may be able to think logically and independently and find rules themselves. This experiential learning may be by doing activities with or without using TLM or items of daily life or playing games or puzzles.

Innovative TLM SOGOL creates opportunities



In this line, measurement is one of the important concepts to be used in daily life and different occupations. However, **research studies** indicate **Measurement** as one of the most challenging areas of mathematics in elementary school **(Clements & Bright, 2003). La**ck of estimation skills, and formula based approach without any actual experience creates problems in understanding measurement concept.

Innovative TLM SOGOL creates opportunities for Estimation length , Perimeter and area, measuring curve etc

C. K. Raju -Towards Equity in Mathematics Education 2. The Indian Rope Trick: Rope vs Compass-Box The rope (or string) is flexible in more ways than one and can be used to do everything that can be done with a compass-box. It can further be used to measure the length of a curved line, impossible with the instruments in a compassbox. This is helpful for the measurement of angles, and the subsequent transition to trigonometry and calculus. The rope is also inexpensive, locally-constructible, eco-friendly, and suited to conditions prevalent in countries like India. Hence, it is a superior replacement for the compass-box.

Usha Menon -The teaching of Place Value - cognitive considerations A counting based, number line using approach as in RME would differ very much from the approaches prevalent in India. Yet experience in the last four years shows that teachers very warmly welcome a teaching aid such as the structured bead string/ ganitmala. 9 This can be understood in terms of the traditional rote practices in which the emphasis is on the chanting of numbers forward and backward as well as on various forms of skip counting. In the traditional practice any form of visual support is given only up to number ten. The development of number sense in the traditional practices can perhaps be understood in terms of the structuring implicit in the number names. Yet these practices can be considered to have the potential to blend with those for developing number sense using the empty number line as a new functional learning system.

Hans Freudenthal(Freudenthal, 1973, pp. 179-194).3

- Hans Freudenthal from Netherlands inspired by a different approach delays the teaching of place value and focuses on the development of number sense. The activities chosen, such as the counting on the hundred bead long ten-structured bead string and the jumps on the empty number line keep the numbers whole without differentiating them on the basis of their place value. This approach is strongly rooted in the conception of Freudenthal about the teaching of numbers. He approached numbers by recognising that there was more than one concept of number, which included counting number, numerosity number and measuring number. He argued very strongly that the numerosity number was ‘mathematically insufficient’, `mathematically unimportant’ and `didactically insufficient’ for the teaching of natural numbers **Counting in groups** in early years is considered as one of the important concept for developing understanding of structured counting. Grouping of 10 is most basic being used in decimal number system. Needs enough experience of grouping of 10 for better understanding of place value.

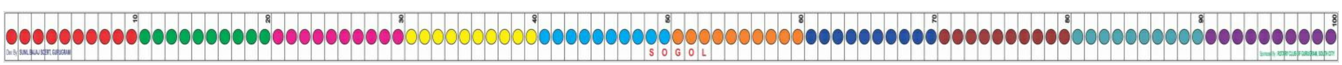
Mathematics is known as study of Patterns. **Patterns** are now introduced at Primary level. As we know finding rules while making wonderful patterns creates foundation for Generalization. While making patterns, children start thinking logically with concentration to find rule. They become interested as they find challenge in it. There is a need of creating opportunities which gives scope for making patterns in interesting way.

Estimation is an area which is rarely discussed in schools. **Larry P. Leutzinger Edward C. Rathmell Tonya D. Urbatsch** in their paper mentioned that Children with experiences of development of **estimation skill** in the primary grades will not only become better at estimating and mental computing but also develop a sense of number. This number sense goes far beyond the ability to compute with paper and pencil. It forms an excellent foundation on which problem-solving and logical-reasoning skills can be based. There is need for providing scope at primary level.

To address these issues, a low cost multipurpose instrument **SOGOL** has been designed which can be used as a TLM . It will help in developing understanding of **Measurement, place value, number concepts, operations in numbers patterns, fractions, decimal fractions, multiples and factors, LCM and HCF and estimation skills.**

SOGOL -An instrument for joyful learning

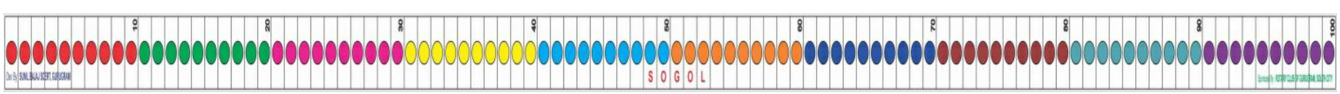
BRIEF ABOUT SOGOL

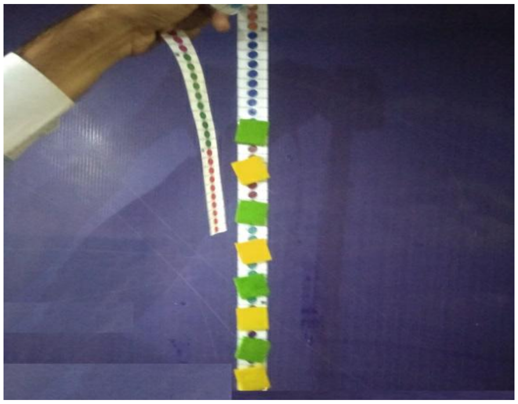


SOGOL

It is a one meter (approx.) long strip (made of flex sheet)with colored dots in groups of ten and each small partition with one dot is of 1cm (approx.) length.

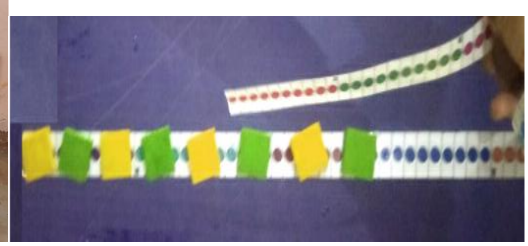
Generally in classroom, One meter measurement tool is rarely available. Children have a vague idea about meter and centimeter actually how much one meter and centimeter is and further 1m =100 cm. Now with Sogol, they can have the idea while using it in the activities on measurement. There estimation skill will improve as they first estimate any length ,e.g., dimensions of Bench, Blackboard then verify with Sogol and will know how much is 1m and 1cm and relation between meter and cm. Also in counting , they have abstract idea of place value. Grouping of 10 in Sogol will help to understand place value and numberness using tiles.





FEATURES OF SOGOL

- It provides the scope of participation of all children as can be easily made available to all children and easy to handle.
- It is a tool for learning by doing. It can be used to measure items of daily life also at home. Helpful in many projects on measurement in and outside class.
- It is used by individual and easy to carry. So can be easily used in classroom to measure length, to find Area and perimeter of notebook, bench etc.



- It can be a measuring instrument and replacement of Geometric box.
- It is durable and can be used for years with small care.
- It is cheaper and will cost around Rs. 6.
- Work like slate as we write with sketch pen or marker , we can rub and clean it.
- Eva tiles stick on it to make patterns so enough scope for inerresting patterns.
- It can be used by teacher to draw an arc or circle and make angle on the black board.

- In the case of the rope vs the compass-box.

  The Sogol as a rope or a string can be used to do a number of things. 1. By holding it tight (possibly by fastening one end) one can draw a straight line, so it can perform the function of a straight edge. 2. By choosing any appropriate unit, it can be made into a scale. 3. By keeping one end fastened and moving the other end around, one can draw a circle. So the Sogol (rope) performs the function of a compass. 4. Most importantly, it can be used to directly measure the length of the arc, hence an angle in radians: simply lay the rope along the curve and note the reading. By measuring circular arcs, it also serves as a protractor which measures angles in radians. 5. By marking two points on it a distance can be picked and carried, so a rope (or string) can perform the function of a divider. 6. It is easy to construct a right angle, and by bisecting or trisecting it, it is easy to construct angles of 45◦ , 30◦ , and 60◦ , so it also performs the function of set squares. 7. By fastening two points, one can also draw an ellipse with the rope. This is impossible with the compass-box. So, string (or a piece of twine) can be used to do everything that can be done with a compass-box, and something

In India, a string with 100 beads popular as Ganit Mala (Usha Menon – Jodogyan) is expensive and is very difficult to handle. Sogol is very cheap. Ganit Mala is mostly used for demonstration big size and is difficult. It is durable and handy so that many games can be played.

CONCEPTS -Counting, grouping, place value, operations, tables, even odd numbers Patterns,

fractions, decimal fractions, Multiples and factors, LCM and HCF, developing estimation and measurement skill ,perimeter and area .

Low cost is going to be as game changer and if used by all and may be made a part of geometry bux/replacement

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ERROR ANALYSIS AS AN EVALUATION TOOL

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As a Mathematics teacher, I was first introduced to the idea of Error Analysis in the year 2015 by Dr Neeraja Raghavan, who was at that time facilitating an Action Research project for conducting multi-age classes for middle school Mathematics. I found the view, that errors can open the window to discovering what the child has understood (rather than serve as a frustrating reminder to what the child hasn't understood), to be a powerful one.

Though a surprisingly simple idea - almost intuitive - my work in this area has led me to see that error analysis can actually cause a shift in perspective and classroom dynamics. It nudges the teacher towards becoming a student herself and coaxes a student to become a reflective thinker. This paper outlines one such journey that a group of students and their teachers embarked upon. It captures the various strategies applied in the Math classroom, related successes and failures and the learning that resulted as we transitioned from analysing errors as an evaluation tool (for the teacher) to analysing errors as a self-evaluation tool (for the student).

INTRODUCTION

Poorna Learning Centre is an inclusive, not-for-profit school located in the metropolitan city of Bangalore. The school has a refreshing attitude towards education, believing that education should be relevant, enjoyable and inclusive. It is a day school, with about 150 students, with a maximum of 15 students per class, 30 teachers and support staff. The school has been supporting children from all socio-economic backgrounds, including first-generation learners. An inclusive community, 10% of each grade comprises students who have special needs.

Though several teachers have provided valuable input over the years, this paper draws chiefly from the experience of two Mathematics teachers who have taught Middle School and High School Mathematics at Poorna Learning Centre. Both teachers, Kanchana Suryakumar and Hemalatha Gowda, have moved from corporate careers to Education. We have six years and ten years of experience teaching High School Mathematics, respectively. We have been actively involved in designing and implementing various strategies to make Mathematics teaching and learning more meaningful and enjoyable in Middle and High School.

The children whose work is referenced in this paper are currently in their tenth grade. This is a group of fourteen students with whom we have been working since their seventh grade. Most of the examples and instances cited in this paper are from their work in the ninth grade. The learning, both teachers' and students', is cumulative and spans three years. This paper is an effort to document this three-year learning.

The errors covered in the examples in this paper include errors in Arithmetic, Algebra and Geometry, at High School level.

ERROR ANALYSIS IN THE MATHEMATICS CLASSROOM

Analysis of errors is sometimes more of an art than a science. It needs an environment that supports an open discussion of errors and a willingness to learn from one's own as well as others' mistakes. It also requires the creation and use of a simple glossary of terms that can help ease communication and convey ideas without confusion. This done, a space is then created for the discussion of strategies to overcome these errors. The paper discusses this process of error analysis in the classroom by fleshing out the details related to:

- Creating an environment for error analysis – various classroom strategies adopted by the teacher to help create an atmosphere conducive to discussing errors without fear or shame.
- Categorisation of recurring errors – grouping "typical" recurring errors into a manageable list of categories. This list is drawn from classroom experiences and review of students' work. It is "work-in-progress" by nature. The list is not meant to be exhaustive, but a starting point to help facilitate classroom communication.
- Assigning errors to appropriate categories – the process adopted by teacher and student to understand the cause for a specific error, which then helps to classify it.
- Strategies proposed for recurring errors – once errors are categorised, the process of evolving and trying out various strategies to address these errors.

OBSERVATIONS

Data for this study was collected through: review of student notebooks, tests, examinations; classroom interactions; self-evaluation forms (written by student) and PTM reports (written by teacher) and one-on-one interviews with the students.

An important takeaway for us teachers has been the realisation that different children take different paths and different periods of time to reach their "zone of comfort" with Mathematics.

While this learning may seem like an obvious conclusion, we, as teachers, needed to practically experience this ourselves in order to come to terms with what the process of teaching-learning entails. To flesh out one aspect of this learning: we have often found ourselves complaining about superficial, or method-based understanding versus a deeper, application-based understanding of the subject. Though some students "got" the problem, we would lament the fact that they did so only superficially. However, as we viewed the progress of children over a longer time-frame, of three-five years, we realised that different children make the journey towards deeper understanding in their own way. While some may start-off with pure pattern-matching and method-based solutions, they add layers to their learning as their foundation becomes stronger. They are able to make the transition to deeper and more intuitive understanding, with practice. The key ingredient in such cases has been **consistent practice**.

Figure 1 captures our observation on the increasing comfort level with Mathematics of the children in our sample set, over a period of three years (with the 2015 values acting as a baseline for comparison). Here, "comfort level" indicates a deeper understanding of the topics and not just mastering exam-writing skills. It has been measured through the following parameters: test/exam performance, clarity in reasoning in the classroom, ability to defend one's solution, ability to make cross-topic connections and ability to teach another student.

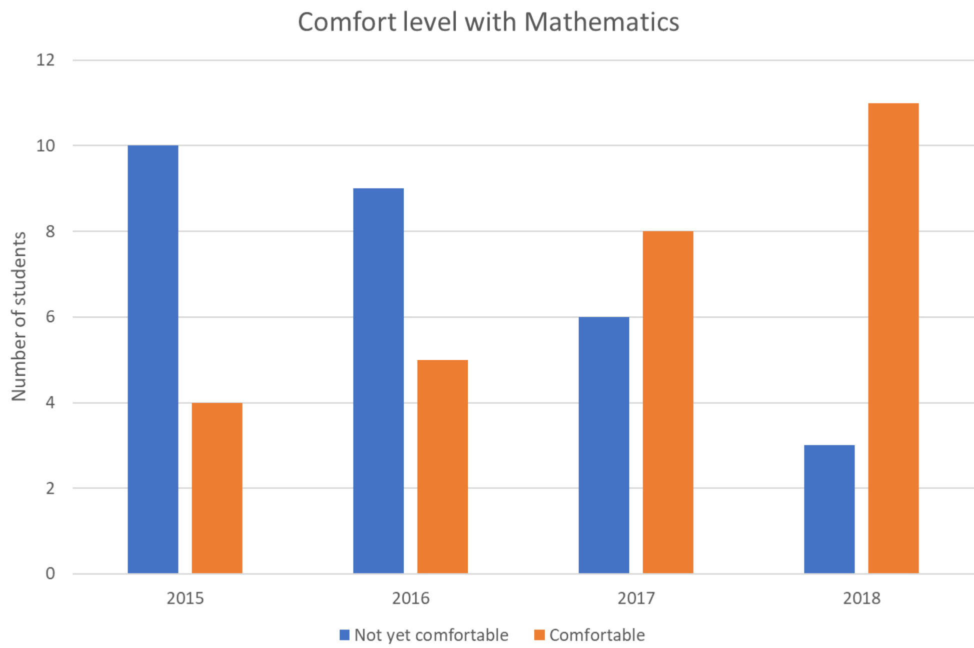

Figure 1 Multi-year progress of students in Mathematics

As part of the study, each student was interviewed to review strategies and processes followed in the classroom and their learning over time. There were two significant observations made by the students.

Firstly, all students said that they did not experience fear or embarrassment while discussing errors. They found it useful to discuss their own and others' mistakes – it helped them learn better.

Secondly, they felt that the classification of errors helped them analyse their own errors and identify the top categories that they needed to work on. Every student was able to analyse his/her own progress over the last few years and talk about their current challenges and their strategies to overcome them. Statements like "I used to make more Copy Errors before, but I now make time for revision and these errors have reduced" were made by each student interviewed.

The students' ability to self-analyse and articulate their views on their own progress reaffirmed our belief that time spent on creating an environment where mistakes can be openly discussed and debated, is time well spent.

REFLECTIONS

As the famous Chinese proverb goes "Give a man a fish and you feed him for a day. Teach a man to fish and you feed him for a lifetime"; instead of just pointing out errors to the students, helping them analyse their own errors has been a very fruitful experience for the teachers. While it is useful to broadly categorise errors and work on common strategies to overcome them, the end goal is not to come up with an exhaustive list of errors and strategies. During the process of interviewing children, what emerged as most satisfying was that they have started analysing their own errors and hence their own progress. The teacher continues to play an important role in pointing out patterns and nudging the child to reflect, but the student has now started playing the more crucial role of a self-learner.

Error Analysis is a long-term process. It requires continuous revision and addition – a process that helps both the teacher and student evolve. It has proved to be an effort well worth investing in.

Dimension Destination: A Digital Game for Secondary School Mathematics and the Sustainable Development Goals

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Introduction

Gone are the days when imparting maximum knowledge and information was the primary objective of formal education systems. With the advent of technology, information is now available at the palm of our hands. The desirable skills for current generation learners are problem solving and critical thinking. These skills are needed for sustainable lifestyles and development of our world. While concepts of sustainable development are challenging to be taught in isolation, children often don't find a connection between the ideas of sustainability and the disciplines they study. This leaves educators with a challenge to embedding these concepts into the disciplines. With this problem in mind, and when talking about developing multi-disciplinary skills, interactive video games can become effective learning tools. While games based learning has already proven its strength from the learners' point of view, the incapability of commercial games to achieve desired learning objectives because of lack of sound educational and pedagogical aspects, it calls for game developers and educationalists to come together. This paper presents one example of a single player, third-person, designed in 3D environment, computer game titled 'Dimension Destination'. It is aimed at students of secondary grades, was built through Unity and focuses on developing mathematical skills while fostering ideas of United Nation's Sustainable Development Goals (SDGs). The specific learning objective behind the game is to reinforce coordinate geometry skills among students, inculcate a sense of spatial awareness and geometric intuition in the player through a context and scenario of sustainable lifestyle and judicious decision making for the health of the planet. The resource opens new domains for further investigation into effective integration of digital games into the curriculum and embedding concepts of peace and sustainable development into games and other digital learning tools.

The Study

Discussing the state of mathematics education, while interdisciplinarity has become relevant for emergent sciences and social sciences of the 21st century, the discipline of mathematics is still rooted in rudimentary computational processes and problems (Sriraman). Students developing an enjoyment and fascination with mathematical concepts has been identified as one of the goals of mathematics education in India (National Council of Educational Research and Training, 2005). A lot of stigma is associated with school mathematics (Swan 2004). Students are hardly able to visualize its applications in real life and often find it boring, which is mostly because of the way it is taught (Payne and Smith, 2010). It is because of these reasons, the researcher explored the possibility of making learning math more fun for students using games based and multi-disciplinary approach. The game was developed by the researcher himself as an attempt to make learning mathematics more fun for students through games based learning, at the same time help students be aware about the SDGs and test the effectiveness of such an approach. The researcher consulted secondary students to learn about what could be an appropriate story and design for the game. The learning objectives of the game were listed and then specific tasks and in-game activities were designed and mapped to the learning outcomes the game is intended to achieve. A short experimental study was also conducted by the researcher to test the effectiveness of the game and that employed a mixed method approach. Quantitative data on tests was collected in form of pre-test and post-test to measure effectiveness of the interventions on learning outcomes. The quantitative and dependent variables measured through the tests were “understanding of the cartesian coordinate system”, “application of coordinate geometry”, “awareness of the SDGs”. These were also the learning outcomes from the game. Qualitative data in terms of students’ opinions about the use of games and what they felt about their learning was collected through interviews after the intervention and written post-test. This included “learner engagement” and “learner motivation” on usage of games based learning. The overall objectives of the game are: to demonstrate understanding of principles and concepts of coordinate geometry, to identify and understand that mathematics can be used and has applications in real life scenarios, to identify and be aware about the United Nations’ Sustainable Development Goals.

About the Game

The game titled ‘Dimension Destination’ was developed specifically to reinforce understanding of coordinate geometry skills and inculcate a sense of spatial awareness in the player. ‘Applying mathematical knowledge and skills to familiar and unfamiliar situations’ has been identified as one of the many overall goals of secondary school mathematics curriculum in curriculum frameworks of many states (Department of Education, 2013; Faculty of Education and Department of Mathematics, 2015; UNESCO 1984; European Commission, 2011). At the same time, these curriculum frameworks for mathematics education also mention inculcating spatial awareness skills as one of the goals of teaching geometry to secondary school students. The game employing these two principles, requires the players to apply their understanding of the 'Cartesian Plane' and the coordinate system, and spatial reasoning skills to move forward and progress in the game. The game begins with a backstory that the player is supposed to help the human race achieve the SDGs by collecting objects associated with different SDGs. The location of these collectibles or objects is provided to the player as coordinates on the Cartesian Plane. As the player moves forward, spatial reasoning is also being inculcated and developed gradually in the player as he/she tries to find collectibles with help of coordinates to their location given to him and trying to identify that with respect to his own position in the environment. Classroom discussions should ideally follow playing of the game to provide opportunities for students to discuss what they have learnt and how they think mathematics can help solve real life problems. This also provides scope for talking about the SDGs, sustainable development and the idea of sustainability. 'Dimension Destination' only runs on computers with Windows platform.

Students are generally attracted towards playing digital, video games and were highly engrossed during the lesson conducted as part of the study. When asked about what they learnt from playing the game during interviews, most students mentioned that they could explicitly see how mathematical concepts might actually work in real life. Using the coordinate system to identify location of collectibles, students were able to identify and map similar real-life applications of coordinate geometry. Some of them suggested that they now were able to visualize how the Global Positioning System (GPS) works and the benefits of having an imaginary coordinate system imprinted on earth, i.e. the latitude and the longitude system. The other observation made by responses from students was that they could acknowledge the seriousness of sustainable lifestyles and living and their own awareness of the SDGs. During the discussions, some of them mentioned going back and reading more about the SDGs, what they could do and their role about them. Feedback given by students as responses in qualitative post-test and discussions depicted high "engagement" and "motivation" to learn through games based approach. 87.5% students said they would want to be engaged again in games based learning sessions for mathematics and also felt that this kind of an activity keeps them motivated to come back and do it again.

Results

From classroom observations, it was seen that even though games can be regarded as standalone tools to promote more non-formal and informal learning, it is important to have well-structured lesson plans and designs for maximum efficiency for classroom use. To have a significant impact and to promote games based learning and usage of more and more games in a classroom setting, it is critical that the usage of games is well planned and integrated properly within the curriculum. The teacher’s role in games-based learning processes is critical and it can be argued that the diffusion of game-based learning can be facilitated only if both learners’ and teachers’ needs and goals are taken into account (Ketamo, 2013). It was observed that without well-organized lesson plans and structure, games also sometime may lead to unnecessary discussions and topics, students losing sight of what is required from them and maintenance of classroom discipline.

This study was conducted to explore how games-based learning can be used to embed concepts of sustainability and the SDGs into the core discipline of mathematics. Games promote systems thinking and a whole brain approach (Francis, 2006) which was the reason to look into the idea of embedding and using inter-disciplinary approach by adopting games as learning tools. It was found that this approach had a positive impact on learner’s performances. Games help students perform in scenario based and problem-solving situations. It is crucial that while developing teaching-learning resources or games for embedding concepts of peace and sustainability into core disciplines, each idea should find its right place in the story and context of the game and that desired skills can be at least explicitly identified if not implicitly. This study involved a short prototype game, and there is a requirement for more research and in-depth review into using games based approach to embed concepts from different disciplines for teaching learning purposes.

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MATCHSTICK GEOMETRY – A CLASSROOM EXPERIMENT[1]

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Introduction

The following write up describes a set of classroom sessions where students explored the geometry of matchstick shapes.

In their journey through numbers, students first encounter counting numbers, then they add 0 to the list, and then integers, the rationals and finally the reals. Students do not have an intuitive understanding of the real number system and its properties (say the denseness property or even the irrationality of $\sqrt{2}$) and these are often taken for granted as having been 'learnt'. Even though the geometry that they first encounter is based on the real number system, the lack of clarity about the real number system does not seem to hamper their geometric journey.

Restricting to shapes that can be constructed by placing matchsticks (or any unit-lengths for that matter) end to end, including at an angle, without bending or breaking units results in a 'new world' where the rules of the game and hence their consequences are different from what they are familiar with.

For example, under the restricting conditions, it would not be possible to

- construct familiar figures like an isosceles right-angled triangle or a circle,
- replicate any angle (as we would be able to using ruler- compass constructions), or
- construct diagonals for all rectangles.

Similarly some theorems about matchstick shapes, say
'In a rhombus whose angles are 60^o, 120^o, 60^o, 120^o, exactly one diagonal is constructible.' or
"Both the diagonals of a rhombus are constructible iff its side-lengths are of the form $n^2 + 1$"
are not true of the corresponding Euclidean shapes. Recognising these differences is not a trivial matter for students. The hands-on experience of trying to make these shapes using matchsticks gives them a feel for what is possible and not possible and intuits them to these differences.

The Experiment

In this experiment, students were expected to explore what shapes can be constructed using matchsticks and what shapes cannot ; what further properties of these shapes can be deduced and how these are similar or different from those of shapes with real nmber lengths.

A class of 12 – 15 students of a Corporation High School in Chennai, mostly girls, participated in the experiment. The classes lasted approximately 7 hours in all.

The specific activities that they went through were directed towards three broad questions –

- the notion of congruence and similarity – how do they apply to stick figures,
- describing stick shapes using mathematical vocabulary without resorting to measurement, leading to notions of 'describability' of shapes and

1 I gratefully acknowledge the support extended by the Headmistress, the Math teacher and the wonderful students of Chennai High School in making this study possible. I also thank Prof. R Ramanujam and Prof K Subramaniam for mentoring me through this project .

- constructibility of shapes under the condition that the only permitted steps of construction are laying sticks end to end.

Examples of the specific activities and the student responses on each of these activities are described below.

Notion of congruence: In this activity students were shown matchstick shapes pasted on paper and asked to replicate them. They were given three kinds of ‘sticks’ - toothpicks, matchsticks and broomsticks broken into pieces of equal lengths. The overarching question for the activity was “when are two shapes the same?”. In course of replicating the shapes, students pointed out that the lengths of the sides, number of sticks per side and the angles involved have to be identical for two shapes to be considered the same. Further a rich discussion ensued where questions like the following were posed and answered.
Should we consider:

- a shape with *two matchsticks per side* to be the ‘same’ as a shape with only *one matchstick per side* – all else being the same?
- A square made with *two matchsticks* per side to be the ‘same’ as a square made with two *toothpicks* per side?
- a square made of *two matchsticks* per side to be the same as a square made of *one stick per side, but twice the length of the initial matchstick* ? (the absolute length of the sides being equal)

With some prompts from the teacher, the students came up with three different criteria for two shapes to be considered the same. Every one agreed that the kind of figures described in question 1 above were *similar,* but not *the same*.

Definition I: Two shapes are the same when they <each side> have same length and same number of matchsticks <on each side> and each match stick has the same length.
Definition II: Two shapes are same when their side lengths are the same.
Definition III: Two shapes are the same when they are made with the same number of sticks on each side, irrespective of their lengths.

This activity involved

- students modifying the familiar concept of congruence under the restricted conditions
- articulating their definitions,
- being sensitised to the possibility of multiple definitions for the same concept and
- noticing how the conclusions as to whether two figures are the same depended on the accepted definition.

Possible extensions for this task like the following were not taken up given the time constraints

- choosing between definitions,
- coming up with and articulating the criteria for the choice,
- exploring if the familiar relations between area and perimeter of congruent/similar shapes hold under these modified definitions (for example, would the area of congruent figures be equal under say definition III, how should unit area be defined if they must be equal?)
- what other relations could be found

Describing shapes: In this activity, one of a pair of students were shown a shape and asked to describe it to the other, as if on a telephone, so as to enable the other person to replicate the shape. Initially the students started with descriptions like ‘make an V shape’, but with rebuttals from the teacher and fellow students, soon realised that the angle of the V needs to be specified. They engaged with the problem and gradually moved to more mathematical vocabulary – like “place a stick perpendicular to the first one” or “Place 4 sticks to form a unit square – starting from the top left corner label the square ABCD and from the mid point of AB...” After some experience in trying to describe shapes, they were asked to consider some figures and decide whether they were ‘describable’ or not. The students realised that specifying the angle was the problem in descriptions and came to some tentative conclusions like figures that involve equilateral triangles or right angles are describable, but other angles are problematic.

Through this activity students saw the need for precise mathematical communication, and gradually moved towards using mathematical vocabulary in their descriptions. They also realised that some shapes could not be easily described without specifying measurements and came up with preliminary criteria for this class of shapes in terms of the angles of the shape.

This task could be further taken ahead by having students refine these criteria then on to verifying and proving them, though this may turn out to be fairly involved. They could also go for ‘minimal descriptions’ by looking for the minimum number of properties that will uniquely determine a shape and also explore the relation between describability and constructibility.

Constructibility of shapes: In this activity students were asked to replicate figures that were *drawn* on the blackboard. Some of these figures involved the diagonal of a square, altitude of an equilateral triangle, etc. After some struggle they realised that it was not possible to construct the diagonal of a unit matchstick square by placing sticks end to end without breaking any sticks.

The initial arguments concerned only ‘unit squares’ and were along the lines that the hypotenuse of the right triangle with sides 1 unit has to be greater than 1, hypotenuse being the longest side of the right triangle. The possibility of constructing a matchstick diagonal for a larger square was not ruled out at this stage (the diagonal could be longer and yet an integer). There were some students who intuitively felt that it was not possible for any square. These students were challenged by others who physically made a square of side 2 matchsticks and managed to fit in a diagonal of 3 matchsticks. In the initial stages, their justification for whether something was possible or not was based on whether they could align matchsticks to form the required shape. Some students intuitively saw the link with Pythagoras theorem and eventually realised that this particular square was violating Pythagoras theorem because 2^2 + 2^2 is not equal to 9 (3^2) Here we see them using mathematics to question the validity of a shape that ‘looked’ feasible.

The students were then asked to explore the possibility of whether this was possible for any square at all and asked to justify their conclusion. Considering a number of examples, they came to the conclusion that twice the square of any number cannot have an integer square-root. Here we see them generalising that any matchstick square cannot have a matchstick diagonal. With some support from the teacher they justified this using the irrationality of $\sqrt{2}$.

This task could be further extended to explore constructibility of other shapes , conditions under which some shapes are constructible (Eg: Only those triangles are constructible the cosine of all 3 angles of which are rational.) leading on to proofs of these claims.

Conclusion

The tasks were intended to encourage students to explore how the geometry of matchstick shapes was similar or different from the familiar Euclidean geometry and to come up with definitions or theorems of matchstick geometry. They aimed to give students a flavour of theory-building in mathematics by engaging with processes like definition, (Mariotti & Fishbein, 1997) structuration and abstraction (Mason, Stephens and Watson,2009; Mason,1989). Mariotti & Fishbein discuss two types of defining in mathematics, defining the fundamental objects and defining a new object through a theorem that states the characteristic property of the object. Mason defines structuration as seeing a relationship as an instantiation of a property and manipulating an already discovered property to deduce further properties.

Through the first task students engaged in redefining the familiar concept of congruence under the restricting conditions, though they did not engage in 'defining' as discussed by Mariotti & Fishbein. They came up with a property of diagonals of matchstick squares and they sought a relationship between the angles present in a figure and its replicability, though this did not get crystallised into a property. They did not get to using an already discovered property to derive further properties. Clearly they engaged in 'mathematical activities' though these cannot be strictly called theory building. Further study is needed to delineate the kinds of mathematical activities that they engaged in, the features of tasks and the role of the teacher in enabling such mathematical activity.

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Title: What does the protagonist of '*The gentle man who taught infinity*' have to say about the teaching-learning of mathematics…?

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"What does the protagonist of '*The gentle man who taught infinity*' have to say about the teaching-learning of mathematics…?" But first, a little bit of the story…

It would be apt to begin with a quote by the American historian, Henry Adams: 'A teacher affects eternity; he can never tell where his influence stops.' My relationship with Mattur Venkatadri Channakeshava, the bespectacled gentleman who taught us math in school, is a lot like that.

Thirty years after I graduated from the Baldwin Boys' High school, Bangalore, I decided to write about him. I felt this deep urge to reconnect with him. I wonder why it took so long. After all, I must have passed by his house in Bangalore many times in all these years. I could have met him as many times as I wanted. I didn't. I guess it is all about waiting for the right day to come.

As Adams says, we end up picking many things from our teachers in school, things which affect us in ways that we can only understand much later. And what we pick up keeps visiting us time and again, in ways we cannot foresee.

I have written about Channa because of two impulses – hope and anger. I believe they are a potent combination.

I'm angry because our schools are letting down our children, day after day and year after year. Survey after survey by the government of the day or those outside the government, shows the abysmal depths we have managed to plumb. We cannot sink any further. The only way is up. The crisis of learning is hidden, eating away at our vitals as a society. Imagine – a child who reaches the eighth grade carries many years of 'learning deficits' in all the subjects she tries to study in school -- what she manages to learn, in effect, is only the equivalent of the third or fourth grade! What a waste of her time! I'm not getting into a longwinded discussion on why this is happening. The undeniable fact is that it is staring at us, challenging us to respond.

I'm hopeful because I have started telling Channa's story. There is much in it that we can learn to make math learning enjoyable. It is now a drudgery for many, with the robotic and mechanical ways of teaching the subject that we have perfected over a long time. Stories like Channa's, I hope, will help us to start moving out of those miserable depths. And out of 'math-phobia,' which afflicts many. At least then, children will look forward to school.

I have chosen to write about an extraordinary teacher, his subject and his craft. I want to spread the word, so that we all – teachers, parents and those interested in education, can learn a thing or two from him. Stuff that we can do and enjoy. And see the difference. I believe this would be a useful and positive thing to do.

I was fortunate to have learnt high school mathematics from Channa. He didn't make me a rank holder or gold medalist in the subject. No, he didn't. On the other hand, I was just an average student of the subject and very nearly failed in it in the ninth grade. He will vouch for it. But he made me love the subject. That is what he wanted all of us to do, even in a school that was conventional and which prided itself on exam results, a school where corporal punishment was rife. So, let me set out the few things he did which I remember to this day.

Channa made us enquire and think. He was always gently nudging, pushing us to figure out things on our own. Second, his teaching was always based on first principles. You never got a sense that he was ad hoc. He taught you to see a pattern in the madness. Third, Channa taught us mathematics because he wanted us to pursue truth and beauty, not mere exam grades. Yes, you heard me right! Never mind if none of us became mathematicians. For me, he made math fascinating. Finally, Channa was a great hit because he was a master storyteller who loved his craft. He *is* the eternal student of the subject who carries within his being a deep understanding of its structure, its mysteries and its paradoxes. These, he happily shared with us. Which is why, I remember his teaching after so many years.

So, we stood on his shoulders and looked beyond, enjoying the mathematical landscapes that were made visible. Beautiful and often startling landscapes.

Channa took us on a roller coaster ride of the world of mathematics. Beyond the syllabus and our sterile textbooks, from Cyclic Numbers to Euclid to Gauss to Bhaskaracharya to Cantor to Euler and the bridges of Konigsberg, to the Barber's Paradox and Fermat's last theorem and the Four-Color Problem, interspersed with discussions on infinity, Channa allowed us to glimpse the real mathematics. These intriguing patterns, problems, mysteries and paradoxes which are the heart of mathematics, he said, need to be understood by us as students to truly appreciate and love the subject. The best part was how they were intimately linked to what we were studying in school. The saddest part was that they were not included in our syllabus. Our curriculum and syllabus makers deprived us of the best parts, year after year. In our examination centered system, the joy of learning and discovery is lost. Channa reminded us that this need not be so, even in a regimented school.

Suddenly, mathematics had a very human story, lots of meaning, and a character of its own. Channa demonstrated how even the most complex ideas of mathematics could be made simple for kids like us. I wanted to know more and more. What if I was a so called average math guy?

So, what does the protagonist of *The gentle man who taught infinity* have to say about the teaching-learning of mathematics…? While reflecting on this question, I would like to first point out that the teacher, his craft and his subject are inseparable. Therefore, the lessons that can be culled out from Channa's story straddle these three elements. I have outlined a few below:

- Engaging with the discipline as a teacher – Channa best exemplifies the aspect of a teacher who loves the subject (this also came from childhood and later influences); being curious about the subject; being a lifelong explorer and student of the subject
- Using the story as a medium of transaction – Channa generated and kept the interest going by using the story. He had a deeply intuitive understanding and appreciation of 'Pedagogical Content Knowledge'
- Using history along with the story – Channa used history as one of the key underpinning elements of his pedagogy. He was as good a history teacher.
- Through the use of story and history, Channa helped us 'locate' the topic. By doing this, he provided the perspective that math has not descended from hell, that there is a human history to it, that there are connected ideas; that there is a process of development of these ideas (messy though, the process may be, full of human elements in it!); that many things taught in the classroom have their own histories of development
- Teaching based on first principles – for Channa, the purpose of using history and the medium of the story was to develop understanding and perspective. Teaching using first principles meant that he was looking at developing understanding and insight, which is a critical result for any learning. Channa nudged, prodded, promoted struggle; he would not just 'give away' the answer and he would often wait to see what happened as we were learning along. Some of the best examples of this 'teaching based on first principles' came from our learning of geometry
- Everyone should come on this journey – no excuses here! The issue of equity was quite central to his scheme of things
- Valuing the child's ability – the *belief* that every child can understand and appreciate deeper stuff – that is why he went beyond…
- Being gentle and being patient…he completely eschewed corporal punishment in a school where it was rife
- Beautiful blackboard usage…how to use the blackboard effectively for communication

Through examples, the presentation elaborates each of the above themes and focuses on what one can learn from this teacher to enrich the teaching-learning of mathematics at school, especially during times where acing the examinations seems to be the preoccupation of schools, teachers and students.

ABOUT THE PRESENTER

Sheshagiri K.M Rao (Giri) works as Education Specialist with UNICEF. He is currently based in Chhattisgarh. Though he began his work as a Mechanical Engineer in 1991, his keen interest in education led him to become a schoolteacher a few years later. Since then, Giri's work in education has spanned a variety of contexts, with local, national and international organizations as well as with government. His professional interests include Early Childhood Development and School Education. Over the years, he has engaged in these fields by working with children, developing educational interventions in diverse contexts, facilitating processes of organizational reflection, doing educational research and writing on contemporary educational issues. A special area of interest is that of chronicling people’s narratives. Other interests include astronomy, singing. Giri is currently working on his second book, which is a work of history located in education.

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Multidimensional approach to design and implementation of quantitative reasoning course

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Key words: Quantitative reasoning, diversity, support, technology, curriculum

Introduction

The School of Liberal studies (SLS) at Azim Premji University started its undergraduate programme in July 2015. The goal of this programme is to create competent graduates who are socially committed, have deep knowledge in a broad spectrum of areas and come from diverse socio-economic backgrounds. We live in a world where numbers and quantitative evidence inform many decisions at the personal level and at the level of society and public policy. Therefore, it is important for all students to have an introduction to the possibilities and usefulness of quantitative reasoning. This necessitates that quantitative reasoning form an integral part of the common curriculum for every student. In this paper we outline our efforts in this direction: to design a curriculum for quantitative reasoning and to provide academic support to handle the diversity of students.

Section 1: Design principles

Quantitative reasoning can be thought of as "the application of basic mathematics skills, such as algebra, to the analysis and interpretation of real-world quantitative information in the context of a discipline or an interdisciplinary problem to draw conclusions that are relevant to students in their daily lives." It is at the intersection of mathematics, and critical thinking in a real- world interdisciplinary context.

Quantitative reasoning is a compulsory course for all first semester undergraduate students at Azim Premji University. The curriculum and content were guided by the following factors:

a) Remove fear of mathematics
b) Be appropriate for a diversity of learners, in terms of attitudes, preparation and discipline
c) Exposure to real life situations where critical thinking and mathematics are required, thereby also making it interdisciplinary, interesting, and relevant.

Pedagogy and assessment:

a) Use of software such as GeoGebra, and Excel, programming languages such as Python, Scratch to relieve the tedium of calculation and invoke algorithmic thinking.
b) Group projects which require students to select topics, read material, write reports and make presentations.
c) Small group tutorials, and continuous assessment through the semester.

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Section 2: Implementation

In each academic year we offered 5 courses, with each course taken by about 25 students. Two of these courses (Calculus and Precalculus) were exclusively for physics majors and the other courses were open to all students. We discuss examples from courses taught by us.

Subsection1: Practical mathematics

The first priority was to create a different atmosphere in which maths is seen as purposeful, logical, linked to the everyday world, and is even enjoyable. The course was designed using a modular approach that is, breaking the course into four modules which had theory, discussion, and practice, each for around a month.

As an example, we detail the module on numbers and counting. This module includes basics of divisibility, prime and modular arithmetic. Starting with the calendar (e.g. calculating on what day of the week was 2 October 1868), one moves on to prediction of eclipses – near commensurability of three different periodicities relating the moon's orbit, leading to the Saros cycle of the ancient Babylonians. The two basic tools are the Euclid algorithm and the related process of expressing, more and more closely, the ratio of two numbers as a continued fraction. The calculations are lightened by using GeoGebra, and the whole exercise acquires an exploratory twist. The next topic is simple coding/decoding based on the modular reciprocal. Students enjoyed cracking the code - finding the decryption key given the encryption key- using the Kuttaka method. The RSA algorithm is an advanced topic, but verification of simple cases by GeoGebra without the full proof is accessible to everyone and connects with their daily experience with online banking, passwords, etc.

Other modules such as trigonometry (bases on surveying) and exponentials and logarithms (applied to EMI, radioactive dating) and probability (which is very rich in good examples) are handled in a similar spirit. Given that this approach is in some ways diametrically opposite to what students have undergone in the 12th standard, the class starts off on a more or less level playing field. We have observed that taking away the premium on recognition of standard patterns, speed in manipulations, and memory helps in attitude change. At the same time, the assessment can emphasise not just reaching the right answer but presenting it in a form which is logical and intelligible.

Subsection 2: Game Theory

Game theory is a topic unfamiliar to entering undergraduates, but it is highly accessible and relevant. Furthermore, it allows students to conduct actual scenarios in class and observe in real time some very profound outcomes. Students then see how these outcomes are relevant in the real world- such as the Cold War, Prisoner's Dilemma, the Monty Hall problem, apart from general ideas about coordination, competition, equilibrium and so on. The students were first introduced to very simple ideas such as in simultaneous games (example: rock, paper, scissors) and in sequential games (example: knots and crosses). Many of these games were played in class so that everyone could participate and watch different strategies and their outcomes. Then more complex ideas such as in Prisoner's Dilemma were introduced. Here students were introduced to incomplete and complete information, and examples such as the stock market were discussed in depth in class. Students were able to engage with ideas of rationality, both in the context of the individual and the group. Playing the ultimatum game helped students understand how human behaviour is deeply affected by ideas of fairness and altruism, and how this often goes against what is rational for an individual. Here, bringing in important references, such as Garret Hardin's "The Tragedy of the Commons", allowed students to see how the framework of game theory can be used to study such issues. Finally, students were introduced to the idea of equilibrium, maximising utility, and probability in game theory. Through all of this, examples from economics (markets, businesses), biology (evolution, cooperation/competition), political science (war, game of chicken) and other fields were used to enrich classroom teaching.

One of the most successful and meaningful assessment forms was the student project. This was a group project where groups were chosen by students themselves. Students had to take up any aspect of Game theory and explain it. Many of them used material from the textbook, some of the groups followed interesting articles. Examples of the projects ranged from Oligopoly, Stag hunt, Incomplete information in literature (Jane Austen), Prey-predator interactions, Poker, Understanding the marriage problem, War strategies, Cold War, Minority game, Newcomb Problem and free will, etc.

Subsection 3: Epidemiology

The course titled “Diseases. Game theory” was designed to introduce students to situations found in epidemiology as well as topics in Game theory. Epidemiology was chosen as the theme as most students find it relevant and interesting. We chose a mix of case studies, invited lectures, tutorials and lectures and projects. We introduced various kinds of ratios and functions that arise when we study disease- occurrence, spread, growth of disease, indicators of health, estimation of magnitude. The class used fermi estimation in various simple situations but the most interesting one that the class worked out to find the time it would take to travel from the UG campus to Sompura gate in 3 years’ time.

Since it is complicated subject where often correlation and causation are confused, students worked on a project, where they formed groups and had to frame a question based on observations. They had to find data to substantiate their guess and then use Excel to find correlation between data sets and reach a conclusion. Each project presentation was different in content but the class could appreciate the intersection of subjects and how mathematics and reasoning helped in each area. Some of the topics were related to specific life style diseases and associated factors, while others related to governance, crime rate, suicide rate etc.

Section 3: Academic support

The maths programme in the summer is part of a larger summer programme and helps students move from school to college learning. The summer programme is offered to students who have some kind of disadvantage, with the objective of helping them settle before the programme begins. We have conducted three summer programmes so far. We take the following steps: initial diagnostic test, identification of student needs, dividing students into groups with different abilities, choosing the appropriate pedagogical tools, and continuous assessment. At the end of the programme, which typically lasts a month, students take the same diagnostic test to identify the progress made. Following this, student files are sent to faculty of respective majors so that faculty can design and continue the support.

The outcome of the summer programme is multi-fold. Students gain confidence in being able to ask questions and to think critically. The students are able to self-reflect and understand their own weaknesses and strengths, which we believe is a first step to taking charge of one’s own learning process. Students are also able to understand the needs of our programme and grow accustomed to the ideas of liberal studies, continuous assessment, and self-directed and independent work. Finally, we feel that such a programme is crucial for any undergraduate institution that is looking to create an inclusive environment and is at the same time attempting to design a rigorous and broad academic curriculum that will adequately prepare young graduates for further education and careers.

In addition to the summer programme, continuous support is offered. We identify students and the kind of support required based on a diagnostic test. We offer small modules that are taught to small groups. Further we try to align the first-year courses in terms of content, assessment and participation grade. This is to help reduce their feeling of being overwhelmed and ensure regular attendance and engagement.

Section 4: Learnings

Based on our conversations, assessments and formal feedback process, we felt that:

a) Students found the topics very interesting, and since all of them were being exposed these topics for the first time, it created more a level-playing field. This was by design, and it addressed the vast difference in student preparation that we encounter in our incoming batches.

b) The mathematical skills required for these courses were very basic (class 8 or so), so all students, even those who hadn't taken maths in class 11 and 12, were able to engage with the course.

c) However, students were challenged considerably when asked to apply simple maths to these news concepts and ideas. This requires higher order capacities that are better addressed in a potential second quantitative reasoning course.

It is a challenge to design courses for a heterogeneous class which has diversity in preparation, majors and attitudes. We are moving in the direction of streaming the first-year quantitative reasoning courses along majors- i.e., designing a course each for the 4 undergraduate degrees we offer, each catering to the needs of that major. However, our overall experience with the quantitative reasoning courses has been very positive. Given an additional course of this type, we would be able to significantly improve the capacities of our students. However, the limited time of three years that we have with the students does not allow us to add another quantitative reasoning course

Appendix – List of instructors and courses

Alex Thomas (Statistics and Society), Asim S (Statistics and Society), Rema Krishnaswamy (Precalculus), Proteep Mallik (Games, symmetry and counting), Rajaram N (Practical Mathematics), Ramchander K (QR with Python), Sri Ram (Creative Computing), Shomen Mukherjee (Games Theory), Richard Fernandes (Calculus), Kripa Gowrishankar (Quantitative methods in biology), Amit Vutha (Quantitative methods in biology), Shantha Bhushan (Game theory, symmetry and counting).

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From Chalk and Talk to Modern Mock

Use of Technology in Modern Teaching, Learning and Evaluation

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ABSTRACT : Technical advancement has played a vital role in development of modern education system. With the advent of internet and different operating system, access of information has become simpler and easier. Various new innovations in teaching learning and evaluation like MOOC's, Virtual laboratory and classroom has been made possible due to growing technology. Certain software's and mobile application systems has helped students to visualize abstract mathematical concept to a better extent and remember them with ease. Not only this such e-learning applications has also proved to be crucial in increasing the interest of new generation students and to connect with them to a wider extent. The aim of this study is to present a survey result (Undergraduate and Postgraduate Level) of awareness about various software's and mobile applications among students and teachers. How such software's and mobile applications has helped students in getting various options for self learning. Also how they are fruitful for teachers in explaining difficult mathematical concepts and computations. The study will further present how certain software's can be efficient and accurate in evaluation at various levels. Innovations required to make mathematics simpler for students of all caliber is discussed.

Key words: Teaching, Learning and Evaluation.

1 Introduction

Mathematics as a subject has been appealing and fascinating to many but also abstruse and ambiguous to some. Thus it becomes a great responsibility for a mathematics teacher to adopt various methods to improve and upgrade their pedagogical skills from time to time. Innovative methods (See, [1], Section 3), like Inducto-Deductive training, Analytical-Synthetic teaching, Play-Way and Laboratory methods have been tested and adopted at school levels. These methods and other teaching aid like charts, manipulative, etc has brought remarkable results. At undergraduate (U.G) and postgraduate (P.G) level, shift of learning maths is more towards personal interest rather than compulsion. Student's come with a different expectation and demand. To meet such demand, involving technology in teaching learning process can be an efficient tool. We encounter a large number of MOOC's platforms which are convenient and effective in self learning. Students can learn a concept in many different ways by viewing online videos on it. Today students have a opportunity to understand abstract concepts and also improve their computational skills by means of software's available in wide variety. Use of smart boards, tablet PC not only helps a teacher in explaining abstract concepts but also in boosting students interest in classroom.

Developing innovative method in evaluation process is the need today. Technology can again help in easy, fast and accurate assessment. Some mobile applications like exam reader, teachers kit, etc. can prove to be useful in this case. These applications are also cheap and easily available. Such applications can help in assessing large group of students and can produce instant results therefore reducing manual labor. By using these applications a teacher can conduct small exams and maintain records easily which is the need in continuous evaluation process adopted by many institution nowadays.

To understand the awareness and availability of these resources among students and teachers an online survey was conducted in different colleges of Mumbai. Teachers and Students responded to their questionnaire, details of which is discussed in next section.

2 Student Survey Report

Students from 10 different institution across Mumbai were surveyed for which 60 responses were recorded. Among 60 students all from science stream and maths as a subject, 50 were from UG ($FY(18) + SY(18) + TY(14)$) and 10 were from PG.

One of the major quest of this survey was to know the preference of a student in clarification of doubts. Out of 60 responses 30 took the help of online videos and notes whereas 30 liked approaching their teachers, thus showing the equal need of online resources for self study.

Students were also asked about their awareness and utility of mathematical software's and mobile applications in learning, details of which is tabulated below:

<table>
<tr><td rowspan="3">Use of Mathematical software in learning</td><td>Level</td><td>No of response</td><td>Yes</td><td>No, But aware</td><td>Unaware</td></tr>
<tr><td>UG</td><td>50</td><td>07</td><td>16</td><td>27</td></tr>
<tr><td>PG</td><td>10</td><td>05</td><td>01</td><td>04</td></tr>
<tr><td rowspan="2">Use of Mobile applications in learning</td><td>UG</td><td>49</td><td>21</td><td>14</td><td>14</td></tr>
<tr><td>PG</td><td>10</td><td>05</td><td>01</td><td>04</td></tr>
</table>

Table 1: Students Response on awareness

The main reason for unawareness of these resources was due to their unavailability.

Since language act as a barrier sometimes in lecture therefore students were also asked about the idea of providing video lectures of their teacher in other local languages on some topics. Out of 59 responses 21 wanted lectures in local languages, 35 in English and 3 students said they may not refer.

Some of the other questions were on having online bridge courses before commencement of a course, availability of digital library in department, Webinar's (online discussion clubs) with faculties and students of other prominent institutions like IIT, IISER, etc and effectiveness of smart board, PPT and other equipments in classroom. The statistics of these responses is given in the table below.

	No of responses	Yes needed	NO	Available
Online Bridge Course	57	46	11	00
Webinar	53	42	3	8
Departmental digital Library	55	24	21	00
Courses on mathematical Software	59	46	13	00

Table 2: Students Response on e-facilities and other resources

The main motivation behind teaching softwares is to establish connection of mathematics with other subjects. Also these soft skills can increase their chances of employability. Another area of concern was to know the impact of virtual classrooms. Out of 58 students 18 connected with their teachers on online platforms for notes and study material, whereas 18 were unaware and 22 were aware but were not using them. Users felt that these platforms are more convenient in getting study material and other information from the teacher.

3 Teacher Survey Report

20 Faculty of mathematics from 14 different institution with experience ranging from 3−15 years were surveyed. Their response on awareness and utility of software and mobile applications is tabulated below:

	No of response	Yes	No, But aware	Unaware
Use of Mathematical software	18	7	09	02
Use of Mobile applications	18	08	05	05
Computer based test platform available	18	06	NO (12)	–

Table 3: Teacher Response on awareness

(I) Statistical data of some questions on teaching resources are:

- Out of 18 responses 14 said the major reason of hesitation among teacher's in using softwares and mobile applications in teaching process was due to lack of adequate training.
- 12 recommended the need of provision for departmental digital library and 08 already had provision.
- 01 used smart boards, tablet PC, PPT and other equipment frequently, 10 used sometimes and 09 used rarely in their lectures.
- Out of 18 responses 11 agreed that online mentor cell can be a effective tool in removing maths fear among students.
- Out of 18 responses 09 felt the need of inducting courses on mathematical software's in the undergraduate curriculum.

(II) Statistical data of some questions on evaluation:

- On Screen marking system (OSM) is used for assessing theory exams of undergraduate and postgraduate from more than 2 years in Mumbai University. A question on its effectiveness was asked to which, out of 17 responses 11 agreed that it is effective as totaling error and chances of leaving a question uncorrected is avoided.
- Out of 17 responses 8 agreed that Mobile applications can bring fast and easy evaluation system.

4 Conclusion

It is evident from the survey that awareness of mobile learning applications is more among students but mathematical software's are less popular as it is not easily available. Also there is a need to induct courses on mathematical software's which can be easily seen from Table 2. Webinar's can be useful in giving more exposure to students and it can help them in self learning by interactions and discussions. Recording Lectures for bridge courses can be a good starting point for teachers in learning and developing MOOC's. From survey report it is evident that OSM is effective and efficient in assessment process. Another way of reducing manual labor in evaluation is by using mobile applications, thus saving lot of teachers time which can be utilized in upgrading pedagogical skills. All these things can be achieved by providing adequate training to the teachers which is available to some extent.

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ON THE SHOULDERS OF THE TECHNOLOGY GIANT

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Technology has become the persuasive buzzword in hard-selling educational institutions and educational packages. Yet, how much thought has gone into the creation of a 'techno-classroom'? Are there clearly defined objectives which justify this step? How are they aligned to curricular expectations and learning objectives to which pedagogical practices are linked? Are there institutional practices and routines through which technology is incorporated in pedagogical practice?

A basic question that needs to be asked is '*When does technology enable the mathematics class*?'

To answer this question, let us look at three reasonable curricular expectations:

- Develop ability to mathematize one's own thought process; for example 'use mathematical modelling for solving realistic societal problems and those encountered in other disciplines using mathematical thinking and interrelationships of mathematical concepts'.
- Develop proofs and justification using deductive reasoning and logical thinking.
- Enjoy engaging with mathematics and develop confidence in mathematical problem solving and problem posing.

A curriculum that addresses these objectives and a classroom that meets these expectations is clearly delivering a good programme of mathematical study.

With such expectations in mind, we move to the next question: How do we select tech-enabled teaching learning materials and teaching aids? Are they self-sufficient and if not, how do we supplement them? This brings us naturally to the lesson plan. Most of the focus in planning lessons using technology devolves to logistical details. At best, the factor of enjoyment, based on the assumption that students are digital natives, is catered to.

Can technology be levered to lay the foundation for students to become problem solvers? Can worksheets and exercises be designed to help students model problems, investigate and justify findings? Do teachers think of how the right questions in a tech enabled investigation push the student's learning to the next level?
The Position Paper on the National Curriculum Framework 2005 talks of the higher aims of teaching mathematics and the importance of formal problem
solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualisation, representation, reasoning and proof, making connections, mathematical communication. How do teachers discriminate between activities that lend to the higher aim of learning mathematics and those that merely pay lip service to innovative teaching practices?

How do teachers ensure that real learning has occurred in the tech-enabled classroom?
A teacher who is a first time user of technology in the classroom seeks to exploit the potential of the technology. Why **is** technology used in the classroom? There are many persuasive reasons – appeal is one of them. While students are attracted to classes in which technology is used, remember that this is a discerning audience. When the computer is used for boring repetitive exercise, they lose interest quickly enough. We would do well to realise that mathematics cannot permeate through the keyboard.

A second reason is availability and convenience of use of technology. Most schools have invested in the infrastructure and having done so, they expect that good use is made of the investment. Unfortunately, the focus of training teachers to use technology in the classroom is on how to use the software and very little effort is spent on training the teacher to work with it more meaningfully. Though the advantages of saving both time and effort is ostensibly used for teachers to plan the development of skills such as problem solving, team-work and collaboration there is, unfortunately, a danger of shifting the focus to the technology rather than the mathematics. Birch (1945) defined the concept of 'functional fixedness'. I see that though the blackboard is no longer a celebrated presence in the classroom, the projector and screen have replaced it. PowerPoint simply makes for slicker presentations and by 'fixing the function of the device' we simply limit the technology, and worse, the lesson. In this paper, I will be focusing on the use of GeoGebra which is an interactive geometry, algebra, and calculus application, intended for teachers and students. GeoGebra is written in Java and thus available for multiple platforms (including, now, the smartphone). Its creator, Markus Hohenwarter, started the project in 2001 together with the help of open-source developers and translators all over the world. Currently, the lead developer of GeoGebra is Michael Borcherds, a secondary maths teacher. Most parts of the GeoGebra program are licensed under GPL and CC-BY-SA[3], making them free software. One of the sites from which it can be downloaded is http://www.geogebra.org/cms/ [i] Though a powerful tool for investigation, many sketches developed by teachers trained in the use of GeoGebra, end up being demonstrations and celebrate the power of technology instead of motivating students to explore, observe, document, reflect, reason and justify.

I then move to the use of technology as a 'path smoother'. Alan Wigley[ii] has described how teachers often design lessons and teaching plans that gently guide students down the path to understanding a concept. Smoothing the path seems like affirming pedagogy, yet it can carry students along to a pre-defined destination without giving them pause to examine it from all angles, confirm their understanding and raise important 'what if' questions. I will give some examples of activities in which this happens with technology in the mathematics classroom.

I will then focus on the importance of Technology Pedagogy Content Knowledge and illustrate with some examples the possibilities in the use of technology in the mathematics classroom. Since I am using GeoGebra to illustrate these points, I will mainly be dealing with examples from Geometry. Some of these will include geometrical constructions and how they can be used not just as recipes for producing desired results but as vehicles to help students question why a certain step is adopted and what happens if it is modified. The well-known theorem based on the angle in a semi-circle is explored with the use of GeoGebra. An example of how GeoGebra can find the fallacy in a proof will be shared.

I will also describe an investigation using paper folding and how the use of technology has enabled students to arrive at and justify exciting findings. Paper folding is also used in combination with GeoGebra to illustrate the concept of a ‘locus’ something that students find very difficult to understand. In this case, the technology is used to visualize and then represent mathematically and this is where Geometry and Algebra will meet.

In short, in this paper I propose to take a brief look at some ways in which technology is currently being used in the classroom, point out some of the inherent pitfalls therein and then suggest ways in which this can be overcome.

NCF 2005 speaks of the importance of inclusion. If teaching focuses more and more on the brilliance of technology to deliver good teaching, then spaces for discrimination will naturally arise along economic divides. If however, the focus is on the pedagogical brilliance of the tech-enabled lesson, then even the simplest and most available technology can align with the vision of NCF. It will then become apparent that it is time for teachers to let the mathematics speak through the technology. Rather than seeing technology as an attention seeking device, I propose that technology is the giant on whose shoulders students can see further in mathematics. In short, the clarities sought in teaching with technology should be illuminating rather than blinding.

Through this paper, I show that it is time to enable the teacher rather than the technology.

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[i] http://www.geogebra.org/cms/

http://en.wikipedia.org/wiki/GeoGebra

[ii] Alan Wigley (Nov, 1992) Mathematics Teacher © ATM 2008 copyright@atm.org.uk for permissions

MISCONCEPTIONS IN CALCULUS DUE TO INTUITION

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Abstract. In mathematics, intuition may inflict misconceptions among students. This article stresses on identifying misconceptions along with potential causes of their origin and on how to resolve them. The article exemplifies this pressing issue via a few illustrations from our classroom experience in calculus. It contends that if the process of identification and resolution of misconceptions is maintained, it is likely to reinforce teaching and learning in calculus, and in mathematics, at large.

1. Introduction

For most people mathematics is the hardest subject in their school. A lack of proper visualization of concepts in calculus may lead learners to a pathetic situation. An intuition is always helpful in solving problems, even a lot of times, it shows a path to prove statements rigorously. Nobody can deny the vital role of intuition in analysis and in mathematics as a whole.

Although intuitions "familarize" us with abstract concepts, it may lead to misconceptions. Tall says, "Visual intuition in mathematics has served us both well and badly ... many fondly held implicit beliefs foundered when analysis was formalized ..." [3]. In fact, shortcomings inherent in our intuitions compel mathematicians to define concepts rigorously which helps one to argue with no ambiguity.

A good teacher provides the students with certain ways to visualize the abstract concepts; such intuitive ideas however may be too naive to be correct. Therefore, it is an urgent need to carefully identify misconceptions that arise among the students due to their prior learning or intuitions; furthermore, it demands assisting students to overcome their misconceptions.

2. Misconceptions

In this section, we present examples of some misconceptions in calculus owing to intuition. Through these examples, we try to describe how teachers can prevent their students from adverse effects of intuition.

Misconception 2.1. *There is no gap/break in the graph of a continuous function $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$.*

The notion of continuity is introduced for a real-valued function whose domain is a subset of $\mathbb{R}$ [2].

The domain of a function is assumed to be only an interval by several people, but it can be any nonempty subset of $\mathbb{R}$. For example, choose $A = \mathbb{N}$ and $f = f_1$ where $f_1(n) = n$. It may surprise a beginner to learn that f_1 is a continuous function.

A visual intuition for a continuous function is that you cannot draw its graph without lifting your pencil. To fix this problem, students may be suggested to observe that a δ-neighborhood of a point in A could be a singleton set (in which case, the point is called an isolated point). The students should be encouraged to apply the ϵ-δ definition to complete the argument precisely. There are continuous functions (like f_1) for which you must lift your pencil! This discussion is also likely to help to tell why the domain of a continuous function must be an interval to guarantee the intermediate value property.

Misconception 2.2. *For a continuous function $f : A \to \mathbb{R}, A \subseteq \mathbb{R}$, if points in A are chosen "closer", their images are "closer".*

Key words and phrases. misconceptions, intuition, calculus, mathematics education.

Several students start believing that for a continuous function, a "very small" variation in the values of x leads to a "very small" variation in their images $f(x)$.

Let $f_2 : (0, 1] \to \mathbb{R}$ be defined by $f_2(x) = 1/x$. The sequence (x_n) where $x_n = 1/n$, is Cauchy, that is, after some instance, any two terms are arbitrarily close. What about their images? Observe $| f_2(x_n) - f_2(x_m) | \geq 1$ for all $n, m \in \mathbb{N}$, $n \neq m$, even though f_2 is a continuous function.

On the other hand, it is okay to say "when the values of x in A are made close to a *fixed given point* c, then their images $f(x)$ go close to $f(c)$." In fact, this is the sequential criterion for the continuity at a point. In this context, the students may be reminded that the ε-δ definition of continuity of function is for a fixed given point c of the domain unlike the definition of "uniform continuity". This discussion can be extended to introduce the notion of uniform continuity and to tell that the Cauchy sequences are preserved under uniform continuous functions.

Misconception 2.3. *For a bounded set $A \subseteq \mathbb{R}$ and a continuous function $f : A \to \mathbb{R}$, if the arc length of f is infinite, then f cannot be uniform continuous.*

Intuitively, a continuous function on a bounded set with an infinite arc length, may appear to have infinitely many oscillations, and this may compel us to think that the function fails to be uniform continuous, as what happens with $\sin(1/x)$ on $(0, 1)$. On the other hand, the function $f_3 : [0, 1] \to \mathbb{R}$, defined by

$$f_3(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0, \end{cases}$$

being continuous on a compact set, is uniform continuous, but surprisingly its arc length is not finite. The discussion may lay a background of "absolutely continuous functions." It may excite the students to know that the absolutely continuous functions are of bounded variation and to observe that f_3 is not of bounded variation on $[0, 1]$.

Misconception 2.4. *If the graph of a function "tries to become parallel" to y-axis, then it is not uniform continuous.*

This misconception may be motivated by functions like $f(x) = 1/x$ on $(0, 1)$ which, as x approaches 0, tries to become parallel to y-axis and grows rapidly. This rapid growth of $f(x)$ forces required δ to be smaller and smaller for a given ϵ, so the infimum of all δ's is zero, and hence there is no δ which works uniformly for all x.

The function $f_4(x) = \sqrt[3]{x}$ tries to become parallel to y-axis, as x approaches zero but still f is uniform continuous.

The students may appreciate this discussion when they learn about Lipschitz functions which are necessarily uniform continuous whereas not every uniform continuous function is Lipschitz.

Misconception 2.5. *If the derivative of a function vanishes identically, then it is a constant function.*

A visual intuition of vanishing of the derivative of a function is that there is no rate of change in it and therefore, it appears to be constant. But this implication is false as the domain of the function may be disconnected. For instance, the derivative of the nonconstant function $f_5 : (0, 1) \cup (2, 3) \to \mathbb{R}$, defined by

$$f_5(x) = \begin{cases} 1 & \text{if } x \in (0, 1), \\ 2 & \text{if } x \in (2, 3), \end{cases}$$

is zero at every point.

This discussion may help instructors to introduce the concepts of locally constant functions and connected sets.

Misconception 2.6. *If the derivative of a function at some point is positive, then the function is increasing at that point.*

Recall that a function is called increasing at a, if it is increasing in an open interval containing a. The derivative at a point is often interpreted as the rate of change at that point which may mislead us to expect the function to be increasing if the derivative is positive thereat. The function

$$f_6(x) = \begin{cases} \frac{x}{2} + x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0, \end{cases}$$

is neither increasing nor decreasing at 0 even though $f_6'(0) = 1/2$. Observe that $f_6'(x)$ is not a continuous function at 0. To overcome this misconception, students may be suggested to consider derivative as the rate of change only if the derivative is continuous.

3. Conclusion

A research study suggests to make students have an experience of visual intuition through "smartly chosen" examples that are cognitive appealing as well as have seeds for understanding the formal subtleties that occur later [3]. It should be greatly appreciated if a teacher enable his/her students connect their intuitive concepts with the formal definition [1]. There should be a balanced approach between rigor and intuition. An excessive inclination towards one of them may incur disinterest or misunderstandings among the students. A teacher should let students think intuitively and should emphasize to make them prove rigorously so that they can conjecure intuitively and validate them rigorously.

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Research based pedagogical tools: An innovative tool at the hands of mathematics teachers

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One of the main aims of teaching mathematics is to mathematize students' thinking. To achieve this, teaching of mathematics needs to be focused towards inculcating critical and mathematical thinking among students. Research based Pedagogical Tools (RBPT) could be an effective and innovative tool to break the monotony of classroom mathematics teaching and help teachers bring elements of inquiry in their teaching learning process. RBPTs are characterized by 5R's. They recognize research, require research, refine research skills, rewards research, and allow students to report research.

The center for excellence in science and mathematics education (CoESME) at Indian Institute of Science Education and Research (IISER) Pune have conducted 21 workshops for Research based pedagogical tools at national and regional level. This paper is the glimpse of those workshop from mathematics view point.

About the workshop:

The aim of the workshops was to introduce and train participants to use a new pedagogical technique - Research-Based Pedagogical Tools (RBPTs) - which can be used for science and mathematics teaching at the undergraduate level. This method focuses on understanding the process and concepts of science and mathematics, rather than memorizing facts- formula and improves critical thinking and research skills among students.

In this series, workshops are conducted at three different levels, Level 1(National Level-trainers from Sheffield Hallam University, UK) where around 150 participants pan India are trained in the core concepts of RBPTs; Level 2 (trainers from Sheffield Hallam University, UK) where 50 selected participants from Level 1 are trained to become trainers themselves;

and Regional level (trainers from the pool of Level 2 participants) where smaller, local groups of teachers in different parts of India are trained to design and use RBPTs. The workshops were of three-and-a-half-day duration. These workshops are fully funded by Department of Biotechnology, Department of Science and Technology, British Council and Ministry of Human Resource development, Government of India.

Over three years, eight Level 1 workshops, two Level 2 (advanced) workshops and 18 regional workshops, to reach close to 2250 teachers across the country have been conducted. From Ropar- Punjab to Passighat- Arunachal Pradesh, Jammu to Chennai and various other parts in Bihar, Chhattisgarh, Meghalaya and so on.

RBPTs- As a process driven strategy

RBPT dealt with more emphasis on student activity with Research driven curriculum which emphasizes students actively undertaking inquiry based learning. The research element in RBPT demands students doing research. The domain knowledge and inquiry sophistication will increase on one hand with more student autonomy. Instead of identifying (What is the answer to this question?), pursuing (What is the answer to my question?) and producing (How can I answer this question?) research, the student should be authoring (How can I answer to my question?) research.

These workshops were truly hands-on, and activity oriented with collaborative learning strategies. There were plenary sessions as well as subject specific sessions which gave ample opportunity for the participants to work in groups and develop their RBPT Posters with discussion and facilitation by the resource persons.

The participants are taken through the process of inquiry with a simple paper plane activity. They get opportunity to explore their research skills. The *Ah-Ha* effect is the summing up of this activity where they actually get to figure out about parameters- variables, hypothesis and the mathematical modelling of the paper plane to fly!

The participants in the group of five are supposed to select a topic they are teaching in undergraduate/ postgraduate level and develop a poster with following components;

1. The Problem to be Solved: Interesting title of the poster, the problem stated clearly- being solved and its context in detail. Giving detailed examples of types of possible

solution with the mathematical explanations, showing the actual mathematical element in it.

2. Activity- The activity to be done by the students, being discussed in detail.
3. Output- It discusses on what will the students produce at the end. Also with what format the student will report their findings.
4. Assessment: How will the work be assessed (form and format) and given credit in the course?
5. Resources: Detail list of all the resource materials and equipment needed for the tasks is reviewed and mentioned.

Along with the above components of RBPT, a learning sequence and a detailed session plan was made by the mathematics group for implementation purpose in their actual classroom. In mathematics subject specific group more focus was given to,

- Posing problems in such a way as to come up with solutions
- Solving problems with hands on techniques rather than just jumping to solution with formula.
- Thinking about, how to solve a problem?
- Finding patterns- instead of using just the formula
- How to create problem: What makes a good problem?

For warm up session the following examples were used, which created the interest and thinking strategies:

1. The handshakes problem
2. Gauss Homework
3. Tower of Brahma
4. Frogs games
5. Unstampable
6. Pizza delivery problem

Assessment in RBPT

The assessment in RBPT has to have the research skills components, which makes it different than the routine assessment of just checking the knowledge aspect. A rubric is supposed to be prepared based on the topic and nature of the RBPTs. Various assessment strategies could be employed for topics taught with RBPTs.

After the RBPT Workshop

All resource materials, including the presentations and sample RBPTs are shared, along with list and contact details of all participants. Participants are also invited to keep in touch with CoESME at IISER Pune and remain involved with RBPT methods and other CoESME activities. Continuous follow-up with the participants is maintained and good platform to share their work is provided.

The number of mathematics teachers applying for the workshop was considerably low as compared to other science subject areas. Total 259 mathematics teachers attended the workshop until now. It was found that the number of mathematics teachers were increasing and also taking interest and active part in developing RBPTs.

In the initial few workshops the participants thought that to develop RBPT for Mathematics subject was the most challenging task. The notion of many answers to one question itself is so difficult to digest for the teachers! To come up with the inquiry elements in mathematics was strenuous task. Later with the understanding the process of RBPTs, it became more clear and feasible even for maths teacher. RBPT focused on the process of mathematics rather than concentration on only the single solution. Through these workshops it was found that using RBPT could help in developing the inquiry process in mathematics which could lead to the initial phase in mathematization of thoughts.

Mathematical Folk Riddles of the Bhojpur Region

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ABSTRACT

Ethanomathematics is, at least partly, a study of the mathematical knowledge embedded in the culture of indigenous, traditional communities. It is a novel branch. For over 15 years now we have worked on uncovering the mathematical knowledge of rural communities in the Bhojpur region of Bihar. We will present mathematical riddles from this region which can serve as examples of Fermi problems, intricate unit conversions, number theory, fractions, algebra, game theory, and integer value programming among others. They reveal a surprisingly complex and rich pattern of mathematical thinking by rural folks who were seemingly illiterate or those who had only elementary education. Most of these problems are in the form of poetry. These problems were known as "***Baithaki***" a word which literally means a group discussion. The word however has a dual meaning: the second meaning comes the verb "*baithana*" or attempt the problems by "trial and error". It was often something which was attempted when the community of elders and children were relaxing after a long day's work and after dinner around a fireplace and under the starry skies. "***Baithaki***" is part of a rich oral culture of story telling, songs etc of this region which is sadly becoming extinct and which urgently needs to be nurtured.

We shall present a representative selection of these mathematical riddles and invite the audience to participate in true "***Baithaki***". spirit.

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Problems relating to pasting lemma for continuity and uniform continuity

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Abstract:

In analysis, the **pasting** or **gluing lemma**, is an important result which says that two continuous functions can be "glued together" to create another continuous function. The lemma is implicit in the use of piecewise functions. Some related problems for pasting lemma and a similar situation for uniform continuity have been discussed in this note.

Definition (Continuity of a function at a point)

Let (X, d_X) and (Y, d_Y) be metric spaces and let $f: X \rightarrow Y$ be a function. Then f is said to be continuous at a point $x_0 \in X$ if and only if whenever a sequence $<x_n>$ in X converge to a point x_0 the sequence $< f(x_n) >$ in Y converge to the point $f(x_0)$.

Remark: A function $f: X \rightarrow Y$ is said to be continuous if it is continuous at every point of X.

Pasting Lemma for continuity

Let (X, d_X) and (Y, d_Y) be metric spaces; let A and B be (non-empty) closed (or open) subsets of X such that $X = A \cup B$; let $f: A \rightarrow Y$ and $g: B \rightarrow Y$ be continuous functions such that $f(x) = g(x)$ for all $x \in A \cap B$; then the function $h: X \rightarrow Y$ defined by:

$$h(x) := \begin{cases} f(x) & if \ \ x \in A \\ g(x) & if \ \ x \in B \end{cases}$$ is also continuous on X.

Now, the question arises: Can we relax the condition on subsets A and B from either both closed (or both open)? Equivalently, can we solve the following problems:

Problem 1: Let $f: R \rightarrow R$, be any function such that $f|_{R-\{0\}}$ and $f|_Q$ are continuous. Then prove that f is continuous.

Problem 2: Let $f: R \rightarrow R$, be any function such that $f|_{R-\{+ \text{ Irrational No}\}}$ and $f|_{R-\{- \text{ Irrational No}\}}$ are continuous . Then Prove that f is continuous.

We show that both the problems 1 and 2 can be easily solved with the help of the following Theorem

Theorem1: Let $X = A \cup B$, where $A \cap B$ is dense set in X . Then $f: X \rightarrow Y$ is continuous if and only if $f|_A$ and $f|_B$ are continuous.

Proof. Let $D = A \cap B$ is dense set in X. let d_n be a sequence in D which converge to $x \in X$. To show that f is continuous, we show that $f(d_n) \to f(x)$.

As $x \in X = A \cup B \Rightarrow x \in A$ or $x \in B$.

Now, $x \in A$ and $f|_A$ is continuous also $d_n \to x \in A \Rightarrow f(d_n) \to f(x)$

Similarly, if $x \in B$ and $f|_B$ is continuous also $d_n \to x \in B \Rightarrow f(d_n) \to f(x)$

Thus, we get if $d_n \to x \Rightarrow f(d_n) \to f(x)$. Hence $f : X \to Y$ is continuous.

Using theorem1, we can how solve both the problems.

Solution for Problem1

Note that $R = (R\text{-}\{0\}) \cup (Q)$ and $(R\text{-}\{0\}) \cap (Q) = Q\text{-}\{0\}$, also $Cl(Q\text{-}\{0\}) = R$

Then the result follows by theorem1

Solution for Problem2

Note that $R = (R\text{-}\{+ \text{ Irrational No}\}) \cup (R\text{-}\{- \text{ Irrational No}\})$ and $(R\text{-}\{+ \text{ Irrational No}\}) \cap (R\text{-}\{- \text{ Irrational No}\}) = Q$, also $Cl(Q) = R$.

Then the result follows by theorem1.

Definition (Uniform Continuity)

Let X and Y are metric spaces with metric d. Let $f : X \to Y$ is uniform continuous if for each $\varepsilon > 0$, $\exists\ \delta = \delta(\varepsilon)$ such that

$$d(x_1, x_2) < \delta \quad \Rightarrow \quad d(f(x_1), f(x_2)) < \varepsilon.$$

Theorem (Uniform Continuity in terms of Sequence)

Let $f : X \to Y$ is uniform continuous if $d((x_n, y_n)) \to 0 \Rightarrow d(f(x_n), f(y_n)) \to 0$, for all sequences $\{x_n\}$ and $\{y_n\}$ in X.

Now, we raise two problems

Problem 3. Let $f : R \to R$, be any function such that $f|_{R\text{-}\{0\}}$ is uniform continuous and $f|_Q$ is continuous. Then prove that f is continuous.

Problem 4. Let $f : R \to R$, be any function such that $f|_{R\text{-}\{+ \text{ Irrational No}\}}$ and $f|_{R\text{-}\{- \text{ Irrational No}\}}$ are both uniform continuous. Then Prove that f is also uniform continuous.

Theorem2: For any function $f : X \to Y$, let D be any arbitrary dense subset of X, then the following conditions are equivalent

a) f is continuous
b) $f|_D$ is continuous and f is continuous at each x in D^C.
c) Whenever $d_n \to x \Rightarrow f(d_n) \to f(x)$ for every sequence $\{d_n\}$ in D.

Theorem3: For any function $f : X \to Y$, let D be any arbitrary dense subset of X, then the following conditions are equivalent

a) f is uniform continuous

b) $d(d_n, y_n) \to 0 \Rightarrow d(f(d_n), f(y_n)) \to 0$ for every sequence d_n in D and y_n in X

c) $f|_D$ is uniform continuous and f is continuous at each $x \in D^C$.

Corollary: Let $X = A \cup B$, where $A \cap B$ is dense set in X . Then $f : X \to Y$ is uniform continuous if and only if $f|_A$ and $f|_B$ are continuous and one of these is uniform continuous.

Remark: The above corollary is the **Pasting Lemma for Uniform continuity**.

Note: Using theorem2 and theorem 3, we can solve the problem3 and problem4.

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Effect of Co-Operative Learning on Students' Achievements in Mathematics at Secondary Level

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Introduction

Lev Vygotsky introduced the thought of social interaction to child learning. Learning is the process, act or experience of acquiring new or modifying and reinforcing existing knowledge. It may involve synthesizing of different types of information. Learning is a very complex activity. In our every activity, learning takes place in different ways and at different levels of consciousness. We know that, individual have their preferred learning styles and people learn in different ways. It may occur as part of education, personal development, schooling or training. Learning results informative effect on the mind, character or physical ability of an individual.

According to Posamentier (2006), "the teaching of mathematics is not about dispensing rules, definition and procedures for students to memorize, but engaging students as active participants through discussion and collaboration among students". "Learning will be more successful if students are given an opportunity to explain or clarify ideas" (Burns, 1990). Therefore, the emphasis on students' involvement in the learning process is the main requisite in the development of education in terms of pedagogy. Broadly, the interactive teaching methods can be grouped into three ways: (i) Whole-class discussion. (ii) Peer partner learning. (iii) Co-operative learning. Thus, the level of learning can be groomed in different ways; one of those is the co-operative system of learning.

Cooperation is the basic characteristic of humankind. Most of our knowledge attitudes and values are formed by discussing and sharing what we know or think about our physical (concrete as well as abstract) and social environment. 'People working together on a common goal can accomplish more than people working alone' is a well-established principle of social psychology. Based on this principle, cooperative learning strategies for use in the classroom have developed and worked out.

Researches indicate that interactive and co-operative learning can produce positive effects on student's achievement.[1] Co-operative learning is an instructional strategy, in which pupil works together in small groups to learn to maximize and gain from each other.[3] Co-operative learning method provides students an opportunity to optimally utilize the limited resources. [4]

Rationale of the Study

The author has a long experience of Mathematics teaching at a school level, she discovered three things: First many students do not like to take mathematics course. They think that mathematics is something that causes stress and they have high anxiety about learning mathematics and trying to succeed. Second, mathematics classrooms foster a competitive atmosphere among students and hence some students have difficulty in expressing their understanding of mathematical concepts in front of their teacher. Third, the students are not adapted to take an active role in learning mathematics. In light of these points, she wanted to find a method of teaching Secondary mathematics classes that would help our students understand and enjoy mathematics. She wants to study whether our students understand and enjoy mathematics more if we try a co-operative learning approach through changing the seating arrangement of the classroom. This study aims to explore whether the co-operative learning activities by modifying classroom seating arrangement i.e. the circular seating arrangement contributed to student's mathematics achievement over the traditional seating arrangement i.e., rows and columns.

Objective of the Study

To study the effect of the co-operative system of learning on the Achievements in Mathematics of X class students.

Research Methodology

This study was both quantitative and qualitative in nature since it focused on examining student's participation while working on co-operative learning activities in circular seating arrangements and the traditional rows and columns seating arrangements as well as its impact on their achievement in mathematics.

Sample

The study took place in Jamia Senior Secondary School, Delhi, India. Two sections of class X were selected randomly for the study. Section A (control group) had 30 students while Section B (experimental group) had 32.

Tools and Techniques

i. Questionnaire
ii. Students' marks of their previous examination i.e., 9^{th} class exam.
iii. Mathematics Achievement Test
iv. Observation schedule

Experimental Design and Procedure

The study was introduced by asking students to respond to a questionnaire, which contain three questions. The first question was aimed to collect data about student's perceptions about their own attributes that could affect participation rate inside the class, for this they asked to rate themselves on a scale of 1–5 as to whether they consider themselves shy or interactive inside the class. The option of two seating arrangements i.e., the rows and columns and the circular layout was provided in the second question. The students were asked to choose which one they would prefer to have in their classes. In the third question, they were asked to write the reasons for their choices in the second question. Based on the conclusion of the questionnaire, the classes were assigned their respective seating arrangements.

Moreover, the researcher collected students' previous years' mathematics score for both the groups (i.e. Section A and Section B) beforehand, to ensure that the two groups had achieved the same levels of mathematics performance.

In the experiment, the students had their classes in two different settings. The first was the regular rows-and-columns classroom for the control group (section A) and the other was a big room in which the students could sit in circles around tables in the experimental group (section B). They were taught the topic "Polynomials" for three weeks. In rows-and-columns classroom, there were three students sitting together on one bench and they were allowed to discuss the problems among themselves. In the class of co-operative learning, she organise the class in small groups (heterogeneous) of five to six members sitting in a circular seating arrangement classroom. In creation of groups, the researcher, have in mind students' performance in Mathematics, as well as their interpersonal relationships. In cooperative classroom, the students work together in the group in a co-operative way to understand the concepts and solve problems based on polynomials. During the treatment, the researcher observes the process and she acts as an observer, adviser, mediator, or tutor that helps, support, guides and solves problems in both the classes.

In the end, students were asked to attempt a thirty-minute achievement test paper. This 20 marks question paper had two sets of questions of 10 marks each. Further, each question was subdivided into five short questions containing two marks each.

Data Analysis

To answer the research questions, descriptive statistics and qualitative analysis of student's responses to the questionnaire, observation during the treatment and the thirty-minute mathematics achievement test paper were used.

It was observed that students were actively involved in finding solutions when they worked in small groups in circular seating arrangement. It was noted in almost all observations that everyone was working the problem on his or her notebook. The researcher observed and heard many phrases discussed in the groups such as, "How did you do it?" "Do you understand ___?" "Do you have any questions, please ask." "Do you know why we did that" "I know what I did wrong here." "We can try it in this way." "It can't be possible." or "Can you explain that to me?" She also observed that students who generally keep silent in the class during the teacher's lectures were asking questions of their partners and in some cases, they offers suggestions for solutions. Although she was constantly observing student group work by taking rounds in the room, the students did not asked her for a solution. They only asked, for a clarification of the problem or enquire whether their answer was correct or not. Most of the students checked their solutions by using trial and error methods and they discussed it.

Marks scored by the students in their previous exam and the mathematics test attempted after the experiment (termed as pre and post-test respectively) were recorded. The data was then; compared between the groups as well as between pre, post-test to see whether classroom-seating arrangements affect their achievement. The data was analysed for individual students as well as an average for both the classes.

Conclusion

In this study, it was observed that the co-operative learning by modifying the classroom seating arrangement from the traditional rows & columns to circle arrangement improved the student's learning capability and understanding of the mathematics subject significantly. This interpretation is mainly based on the comparison of improvement in the passing percentage and the marks obtained by the students of the both the groups (Section A & B) in the post-test mathematics achievement test paper.

Results of this study show that co-operative learning help to improve not only the understanding of the students but also motivate them to discuss the difficulties during the study. There is no other way of the co-operative learning than the groups' study and one way of that is to transform the classroom seating arrangement in a way which suits the most to the requirement of the student's performance. Here the author has shown the modification of the rows & columns into the circle seating arrangement, but there can be many more ways to do so. Some of the students who considered themselves shy but when they seated in a circle, their performance in the achievement test was very good. This means that it could be claimed that co-operative learning through class seating arrangements, not only affect the highly interactive students in the class but could also help shy students to be more active and participate in the discussion which in turns improve their performance in the subject.

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विद्यार्थ्यांमध्ये गणिताबददल आवड निर्माण करणे

श्री. सुभाष उत्त्म भोई

वेलीमाता माध्यमिक आश्रमशाळा वेली.

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आमची शाळा आदिवासी अतिदुर्गम भागात आहे.या भागातील विद्यार्थी कमालीचे लाजाळू व कमी बोलणारी असतात.अशा वातावरणातून आलेल्या विद्यार्थ्यांना बोलते करणे कमालीचे कठीण काम आहे. यासाठी आम्ही आमच्या पातळीवर,या विद्यार्थ्यांचा आत्मविश्वास कसा वाढेल व त्यांच्यातील न्यूनगंड कसा कमी होइल व गणितात प्रगती कशी करतील यासाठी प्रयत्न करीत असतो.

गणिताच्या नावाने ब-याच विद्यार्थ्यांना भिती वाटते. कारण यातील अमूर्त संकल्पना. विद्यार्थ्यांना एखादी संकल्पना सांगितली की तिची प्रतिमा त्यांच्या डोळयासमोर निर्माण होत असते. परंतू गणिताच्या बाबतीत ही प्रतिमा नेमकी अशी तयार होत नाही. तसेच गणिताचा उपयोग कोठे व कसा होतो हे विद्यार्थ्यांना उमजत नाही. अश्या ब-याच संकल्पना कार्डांच्या,गणिती साहित्याच्या सहाय्याने समजावून देणे सोपे व गतिमान ठरते. असे पर्याय शोधले जातात व गणितातील वेग वाढणे व त्यांच्यातील न्यूनगंड कमी करणे या कडे लक्ष् दिले जाते. या सर्व उपक्रमात प्रगती करणा-या विद्यार्थ्यांना विविध परीक्षांना बसविले जाते.

कार्डे व गणिती साहित्याचा उपयोग

इयत्ता 8वी च्या वर्गात विस्तार सुत्रांची कार्डे वाटुन द्यावीत. शिक्षकांजवळ प्रश्नाची कार्ड व मुलांकडे उत्तरांची कार्डे असतील. शिक्ष्क एक एक प्रश्न विचारतील, ज्या विद्यार्थ्याकडे उत्तराचे कार्ड असेल तो विद्यार्थी त्या प्रश्नाचे उत्त्र फळयावर लिहिल. तसेच काही विस्तार सुत्रे जीओ बोर्डचा वापर करून समजावता येते. त्याच प्रमाणे एखादे मोठे उदाहरण समजावून देण्यासाठी कार्डांच्या माध्यमातून समजावून दिल्या जाते. कार्डांच्या माध्यमातून त्रिकोणाची एकरूपता, समरूपता यांच्या कसोटया समजावल्या जातात. करणीचे नियम समजावले जातात.

विद्यार्थी या कार्डांवरून अधिक सराव करतात. आक्रूत्यांचे विविध प्रकार जीओबोर्डच्या सहाय्याने दर्शवता येतात. त्यावरून ते आक्रुत्यांचे गुणधर्म सांगतात. तसेच जीओबोर्डवर स्तंभालेख तयार करणे हे ही सांगितले जाते.

प्रश्न उत्त्र कार्डाच्या उपयोगाने, उदा. एका कार्डाच्या अर्ध्याभागावर प्रथम विचारल्या जाणा-या प्रश्नाचे उत्त्र व अर्ध्याभागावर पुढील प्रश्न असतो. प्रथम कोणत्याही कार्डावरील प्रश्न वाचून त्याचे उत्त्र असलेले कार्ड विद्यार्थ्यांकडून मागवायचे व सर्व कार्ड एका रांगेत मांडायची. हिच कार्डे अदलाबदल करून ही क्रीया वर्गात करून घ्यावी.

आमच्या शाळेत गणितावर आधारीत स्पर्धा ठेवल्या जातात. यातून चांगले विद्यार्थी निवडले जातात. तसेच वर्गा वर्गात प्रश्नमंजुषांचा कार्यक्रम घेतला जातो. गणिती खेळ,स्क्वीज घेतल्या जातात. या सर्वांमधून जे गुणसंपन्न् विद्यार्थी असतात अश्या विद्यार्थ्यांना निवडले जाते. आणि अशा प्रकारे आमच्या आदिवासी विद्यार्थ्यांमध्ये गणिता बददल आवड निर्माण केली जाते.

ABSTRACT

TITLE: PALLANKUZHI

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DECIMAL NUMBERS

Addition and Subtraction in Decimal numbers

Introduction:

"Some Mathematician, I believe has said that true pleasure lies not in the discovery of the truth but in the search for it" – Tolstoy

"Mathematics is the queen of science and Arithmetic's is the queen of mathematics "– Carl Friedrich gauss

We feel pleasure with such golden quotes. But at the same time we are struggling some time to make the children to understand the subject.

Some statistics show there is a lack of knowledge in mathematics. It is a bitter truth

As a maths teacher we have some responsibilities to rectify this problem and should make mathematics as one of the lovable subjects.

Problem faced

When teaching the topic decimal numbers addition and subtraction in standard VII the main mistake committed by the student was writing the decimal numbers in wrong method. They made mistake when writing the numbers one below another

A Play way method be used to rectify the problem.

Our School is situated at a small village. In our village now itself we can see many traditional games like Thayam, Pandy,kili thattu,Pallankuzhi….etc

We have used **pallankuzhi** to do decimal number's addition and subtraction.

Pallankuzhi:

Pallankuzhi is a traditional ancient Tamil mancala game played in south India especially in Tamilnadu. Later the games were spread to other state - countries like Karnataka, Andrapradesh, Kerala, and Srilanka & Malaysia.

Figure 1 Pallankuzhi Board

Players:2

Skills developed: Number system, Counting skill, decision making skill, arithmetic calculation, Memory skill, thinking skill..,
All age group people & children play this game especially girls play it.
We have used such a ancient board pallankuzhi in decimal numbers 'Addition' & 'Subtraction'.

CONTENT:

Decimal numbers definition:

A decimal number is a number with a decimal point in it. The number to the left of the decimal point is an ordinary whole number. The first number to the right of the decimal is the number of tenths (1/10s) the second is the number of hundredths (1/100s) and so on.

Decimal Point - Definition:

Decimal point is a point or dot used to separate the whole number part from the fractional part of the number.

Example 3 7 $\boxed{\cdot}$ 4 2

Whole number Separator Fractional part

In Decimal numbers, decimal point is act as a **indicator** which is whole number part and fractional part.**(Separator).**

it doesn't have any place value.

Required things:

Pallankuzhi board, place value chart strip, small size number cards (single number in one card),addition and subtraction symbol cards, card strips of decimal numbers, tamarind seeds.

ACTIVITY:

Figure :2 Usage of Pallankuzhi in Decimal numbers

Taking any two decimal numbers cards.(eg 43.52 & 15.6)

For the first number, taking the individual number cards and the decimal point card that is 4,3,.,5,2 Placing the cards from left side to right side of the pallanguzhi board .

For the second decimal number 15.6, we have to take 1,5,.,6 (single cards)

Practicing the pupils to put the decimal point card first, just below the decimal point card in the upper row. Asking them to place the other single number cards right and left side of the decimal point card.

Practicing few times, placing the cards in this method we can make the students to put the decimal numbers in correct place. By practicing more and more they can easily understand the concept. This is the main task.

After that asking them to put the tamarind seeds in the pallankuzhi pits where we placed the number cards in corresponding number values.

We have to put together the tamarind seeds of each column from right side to left side one by one

When the total number of seeds exceeds 9 (nine) it will be two digit number) suppose it is 12 we put 2 (two) seeds below in the corresponding column

Then 1 seed be placed at the top of the next column (left side)

When adding the seeds in the second column, we should add the seed from 1st column.

This method is continued …

Then the common method of **addition** be followed, to get the answers.

The same method can be used for decimal numbers **subtraction.**

CONCLUSION

This play way method is very useful for slow learners and below average students .It is based on learning by doing methods it remains in our mind long time. At this same time the children know our traditional game board **pallankuzhi** and it's playing method. It may be one of the ways to safeguard our ancient games

Teaching Calculus to Management Students

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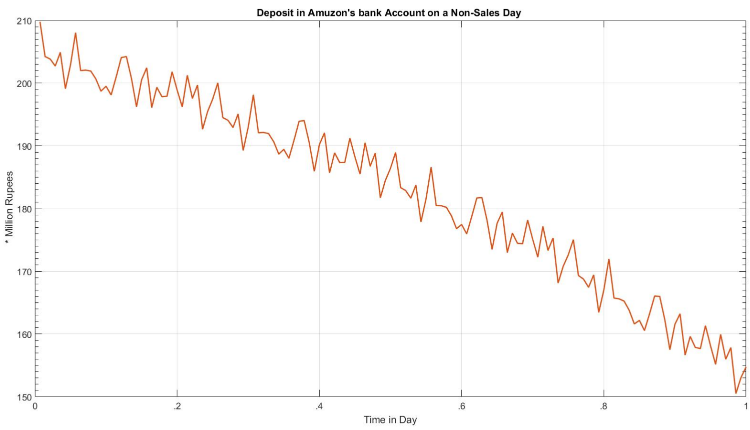
When it comes to teaching Mathematics to undergraduate students, the pedagogy differs with the audience. I have been teaching Calculus in the Integrated programme in Management at IIM Indore for the past 4 years and have been trying out different ways of introducing concepts of Calculus.

In this poster, a different technique of introducing Riemann Integration as a limit of sums, will be presented. Generally, the concept is introduced as area under the curve of the function $y = x^2, x \in [0, 1]$. The region is shown to covered by a finite number of rectangles, one set "over-estimating" the area of region and another by "under-estimating" the area of the region. In this paper, we do not follow this approach and present a way to introduce integration through a scenario of bank account's transactions over a business day.

Assume that the following graph shows the deposit in the bank account of a company named Amuzon which deals in online sales of garments on a non–sales day.



Amuzon has an active bank account and has deposits, running in lakhs, each day. However, Amuzon is not happy with the current interest policies of the bank and has expressed the same to the bank authorities. Since the bank does not want to loose out on such an active account, it has been giving a thought on ways to provide Amuzon with a better interest policy. Some of the business analysts of the bank have proposed that they should form a policy which captures every transaction of the day and not biased towards maximum or minimum deposit during the day. The team sits down to solve the problem.

They wonder that if the principal is constant throughout the day, then the interest is nothing but the simple interest PR, where P is the principal and R is the rate of interest. Now, clearly, the principal changes each instant and hence this might not be the best way to do. Infact, the bank is calculating interest on minimum amount of the day at present, which is clearly impractical for Amuzon. On the other hand, if the bank starts calculating interest on the maximum amount of the day, it is not only a loss for the bank, but can also leads to frauds at later stages. So, what to do if the principal is a non-constant function of time?

They aim to capture the complete trend, but it is not defined which instants of the day to pick up for calculating the interest. So, to start with, it is advised to pick up n, uniformly spaced instants of the day.

To that end, it is assumed that the day is given by the interval $[a, b], a, b$ are real numbers. Let

$$f : [a, b] \to \mathbf{R}$$

such that $f(t)$ gives the money in the bank account at the time t. The time interval $[a, b]$ is then divided into n equal sub-intervals, i.e.

$$[a, b] = [t_0, t_1] \cup [t_1, t_2] \cup [t_2, t_3] .... \cup [t_{n-1}, t_n],$$

where

1. $$t_n = b, t_0 = a$$

2. $$t_k - t_{k-1} = \Delta t, k = 1, ..., n$$

3. $$t_k = t_{k-1} + \Delta t, k = 1, ..., n$$

Hence, $t_n = t_0 + n\Delta t$ which implies that

$$\Delta t = \frac{t_n - t_0}{n} = \frac{b - a}{n},$$

and

$$t_k = t_0 + k\Delta t = a + k\frac{b - a}{n}, k = 1, 2, ..., n$$

Let P_n denote the collection of time points

$$\{t_0, t_1, t_2, ..., t_k, ..., t_n\}$$

The aim to calculate the interest earned in each time interval $[t_{k-1}, t_k]$ and add them up to hopefully estimate the interest over the whole day $[a, b]$. The interest is nothing, but the principal in the interval $[t_{k-1}, t_k]$ multiplied by the time duration multiplied by rate of interest. But, since principal is a function of time, it varies over $[t_{k-1}, t_k]$. Let the rate of interest is 100% and $\overline{t_k} \in [t_{k-1}, t_k]$ be any time point in $[t_{k-1}, t_k]$. Assuming, that principal is constant on $[t_{k-1}, t_k]$ and given by $f(\overline{t_k}$, then the interest in the time interval $[t_{k-1}, t_k]$ is given by

$$f(\overline{t_k})(t_k - t_{k-1}) = f(\overline{t_k})\Delta t, \ \ k = 1, ...n$$

Note that there are infinitely many choices for $\overline{t_k}$. The total interest calculated during the day $[a,b]$ is given by sum of all the interests earned in the n sub-intervals. Let it be denoted by $S(P_n, f)$. Then,

$$S(P_n, f) \quad = \quad \Delta t(f(\overline{t_1}) + f(\overline{t_2}) + ... + f(\overline{t_n})) = \sum_{k=1}^{n} f(\overline{t_k})\Delta t \tag{0.1}$$

Note that at each time sub interval $[t_{k-1}, t_k]$, the principal $f(\overline{t_k})$ always lies between minimum and maximum value of principal f in that time interval. Hence,

$$\min_{[t_{k-1},t_k]} f \leq f(\overline{t_k}) \leq \max_{[t_{k-1},t_k]} f, k = 1, ..., n \tag{0.2}$$

where $\min\limits_{[t_{k-1},t_k]} f$ and $\max\limits_{[t_{k-1},t_k]} f$ denotes the minimum and maximum of f on the interval $[t_{k-1}, t_k]$ respectively. Let $m_k = \min\limits_{[t_{k-1},t_k]} f$ and $M_k = \max\limits_{[t_{k-1},t_k]} f$. Then, adding the equation (0.2) for $k = 1., ...n$ and using (0.1),

$$\sum_{k=1}^{n} m_k \Delta t \leq S(P_n, f) \leq \sum_{k=1}^{n} M_k \Delta t \tag{0.3}$$

Let

$$L(P_n, f) := \sum_{k=1}^{n} m_k \Delta t$$

and

$$U(P_n, f) = \sum_{k=1}^{n} M_k \Delta t$$

Note that $L(P_n, f), U(P_n, f)$ and $S(P_n, f)$ are functions of n. Now, since the aim is to capture each instant,

$$\lim_{n\to\infty} S(P_n, f)$$

will help them capture the whole trend and calculate the interest in an optimal way. Now, if

$$\lim_{n\to\infty} L(P_n, f) = \lim_{n\to\infty} U(P_n, f) = L,$$

then using Sandwich theorem,

$$\lim_{n\to\infty} S(P_n, f)$$

will exist and will be equal to L. L, if exists, is defined to be the interest earned on the time $[a,b]$ and denoted by

$$\int_a^b f(x)dx$$

and called as the definite integral of f on the interval $[a,b]$.

Post this, using Excel, the students are asked to approximate the interest using various values of n and various choices of $\overline{t_k}$. Then, definite integrals of various polynomial functions are also calculated using the previous definitions before giving formal definitions of Lower and Upper Sums, Partitions and Riemann Sums.

On Poetic Presentation of Mathematics

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Training of any subject in a musical way add an attractive advantage to the teaching and learning practice. The wide use of music for training of our mind is still an unexplored area of work. This article explores the mystical connection among music, maths and musical way of teaching mathematics with some new examples. The human society has been trying to use music in learning of mathematics since ages. In [3] one can read about the poetic connection of mathematical activities of Sumarian, Akkadian and Babylonian civilization. Everyone's mathematical education follows the path taken by our ancestors. This path starts from the practice of nursery rhymes and learning about the numbers from rhyming words. In the ancient Indian culture during the middle age there are so many evidence of the presence of mathematical concepts in form of verse [2, 4].

The power of poetry and rhyming create spacial attention to the abstract concepts. Poetic lines on 350 years old quest of Fermat by Ted Munger in [5] is very interesting. It creates a lot of quick attention when we use these poetic style of presentation of content in classroom teaching. Following are lines of this poem in which we can see the mathematical humour of Fermat's theorem,

Fermat's Last Theorem Poetry Challenge, Two Limericks

With an integer greater than 2
It's something one simply can't do.
If this margin were fat,
I'd show you all that,
But it's not, so the proof is on you!

Riemann Conjecture is one of the famous problem listed in seven most important open problem by Clay Mathematics Institute. Some of lines from a limerick written by Tom Apostol are as follows,

Where Are the Zeros of Zeta of s?

Where are the zeros of zeta of s?
G.F.B. Riemann has made a good guess;
They're all on the critical line, saith he,
And their density's one over 2pi log t.

This statement of Riemann's has been like a trigger
And many good men, with vim and with vigor,
Have attempted to find, with mathematical rigor,
What happens to zeta as mod t gets bigger.

Extending this particular poetic technique to present mathematics, we have written more then twenty poems based on the mathematics of numbers, geometrical objects, algebraic expressions etc. Each poem have more then fifteen lines containing fundamental concepts with their recent and ancient usability. After adopting this rhyming of the mathematical contents we found significant success in terms of size of content delivered in class and memorization of the fact with example. Judicious selection of poems and careful discussion of content mainly presents two pedagogical benefits, a better integration of material and easier transition to its applications. We hope that this poetic methodology to teach mathematics may engage every student in the classroom at their best. It motivates student to create to construct their own poem on various concepts or techniques in mathematics. Example of these type of project to motivate student can found in [2, 9]. Following are four poems based on the mathematics of real numbers, Circle, Carmichael Number and Heron's formula.

Real Number

Division of integers is a very interesting problem,
We can solve it by Euclid division algorithm.
Euclid division algorithm computes highest common factor,
In this we apply Euclid division lemma till we get zero reminder.
Factorization of an integer is remain a challenge for mathematicians,
Every composite number have prime factors with unique factorisation.
Fundamental Theorem of Arithmetic is crucial for integers,
In Euclid's Elements it was recorded earlier.
Prime factorization is useful to find LCM and HCF ,
HCF times LCM of two numbers is the product of numbers themselves.
Locating irrationals on number line shows beautiful geometry,
For a prime p, It is interesting to prove $\sqrt{p}$'s irrationality.
Irrational numbers characterize by its decimal expansion,
Decimal expansion of rational terminates or non-terminating repetition.
Cantor worked on enumeration of real and rational numbers,
Combinatorics have so many reward-able conjectures.

Circles

Circle attracts because of its symmetrical joint,
It is equidistant from a fixed point.
Euclid's Element was the finest creation on geometry,
Line passing through the centre is called line of symmetry.
Width or length of circle is called its diameter,
Half of diameter is radius, which is a useful parameter.
A circle divides its plane into three parts,
Drawing it from three points is an interesting start.
Try to fix the ration of area of circle and radius squire,
Ratio of perimeter and diameter is also our ancient desire.
A tangent touches circle at the unique point,
Radius and tangent perpendicularly meets at this unique joint.
Tangents drawn from any one external point of circle are equal,
They are adjacent sides of a cyclic quadrilateral.
To fit a circle in squire is an ancient quadrature problem,
It shows π irrationality and useful in many theorem.
Chord and curve of circle cover segment area,
Radius and curve of circle decides sector's criteria.

Carmichael Number

Primes are those which pass AKS primality test,
For pseudo-primes Carmichael raised the zest.

Integer factorization have some fundamental association,
Any prime factor of a Carmichael number have no repetition.
The search of pseudo-primes have various algorithms,
Because of RSA, Carmichael numbers obtain the spatial attention.
It has been proved that all Carmichael numbers are odd,
The congruence relation makes this picture broad.
Alfrod *et al.* proved they are infinitely many one,
Finding of smallest such number was a milestone.
At least three primes are always their factors,
The third Carmichael number is Hardy-Ramanujan number.
Their intimacy with prime is wondering Cryptographers,
Unlocking their secrets can open new prime chapters.

Heron's Formula

Area of a triangle is half of base multiplied by hight,
Major hight in every triangular shape is difficult fight.
Heron's was a famous mathematician from Egypt,
There are various results of mensuration written in his manuscript.
He related sides of triangle with its area,
Proof of this relation came from Pythagoras criterion.
Archimedes may knew this over two centuries earlier,
In some case Brahmagupta and Bretschneider rose the portire.
Heron's gave approximation rule of squire root of non negative number,
Babylonian had its method with another nomenclature.

We can see that conversion of mathematical concepts into verse reduces the size of the content and make them easy to memorize. There are some international level poetry competition organizes by some renowned mathematics agency. Teacher can motivates students to participate in this poetry writing competition related to mathematics [10, 11]. In fact they can organize these competition on classroom level and enhance the creativity of students by these hands-on activity. As we know the main motto of teaching mathematics is to develop the innovative and artistic skills in student. The use of presented poems and poetry practice in classroom can complete it in a entertaining way.

I hope that my work presented here will motivate the group of mathematicians, teachers and students to build their own poems based on mathematical concepts and use them in the experiment of teaching as an innovative tools of teaching.

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Title **: Use of ICT to Teach/Learn Non Routine Problems of Geometry by interactive ways.**

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Abstract

GeoGebra is a dynamic and interactive geometry, algebra, and calculus applications, intended for the teachers and students. Even it is also very useful for statistics and physics.

By the use of GeoGebra, we tried to verify some of the Non-routine Problems of Geometry. This open source software is very helpful to solve some of tedious problems of Geometry. We are going to discuss here five problems of Geometry by using GeoGebra which are little bit difficult for the school students and teachers.

We know mathematics is one of the most important subjects at all level. If anyone wants to learn well Mathematics then he has to prepare himself from primary level. Mathematics is not just solving the problems but finding different way to solve and use them in different situation of life too.

यथा शिखा मयूराणां
नागानां मणयो यथा।
तद्वद्वेदाङ्गशास्त्राणां
गणितं मूर्धनि स्थितम्।।

(Vedānga Jyotisa, 4)

#1

We were working on low cost and no cost teaching aids of Mathematics. Since 1982, we had a mathematics lab in my school (i.e. KV-1, Bhopal in 1982). We were regularly innovating some new ways to teach mathematics in interesting ways and it is continue till date.

Now-a-day, ICT plays important role to teach and learn Mathematics.

We have prepared hundreds of interactive applets on teaching and learning Geometry at secondary level and beyond. These applets are dynamic easy to verify the facts practically.

We will discuss with the worthy participants and experts and show some of the Geometrical Concepts new **prepositions**[#2] as listed on the next page by using GeoGebra :

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1. ABCD is a quadrilateral. M and N are the mid points of its diagonals. Side BC and AD are non parallel. AD and BC are extended and intersect each other at point P. Join Point MNP**.** Show that ar.(ABCD) = 4*ar.(MNP)

2. In ABC, D is any point on side AC. E and F are mid pints of BD and AC respectively. Verify ar.(ABC) = 4*(CEF)

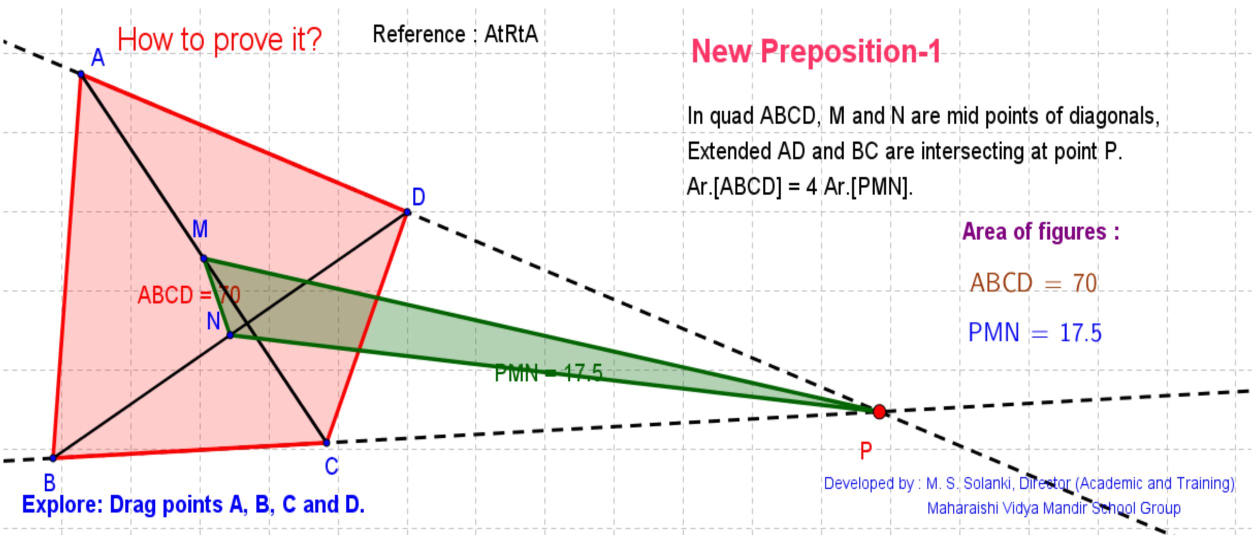
3. In ABC, D is any point on the **extended side AC**. E and F are mid pints of BD and AC respectively. Verify ar.(ABC) = 4*(CEF)

4. In ABC, BY is an **internal bisector** of B and AX and CZ are external bisectors of angles A and C as shown in figure. Verify bisectors AX, BY and CZ are concurrent.

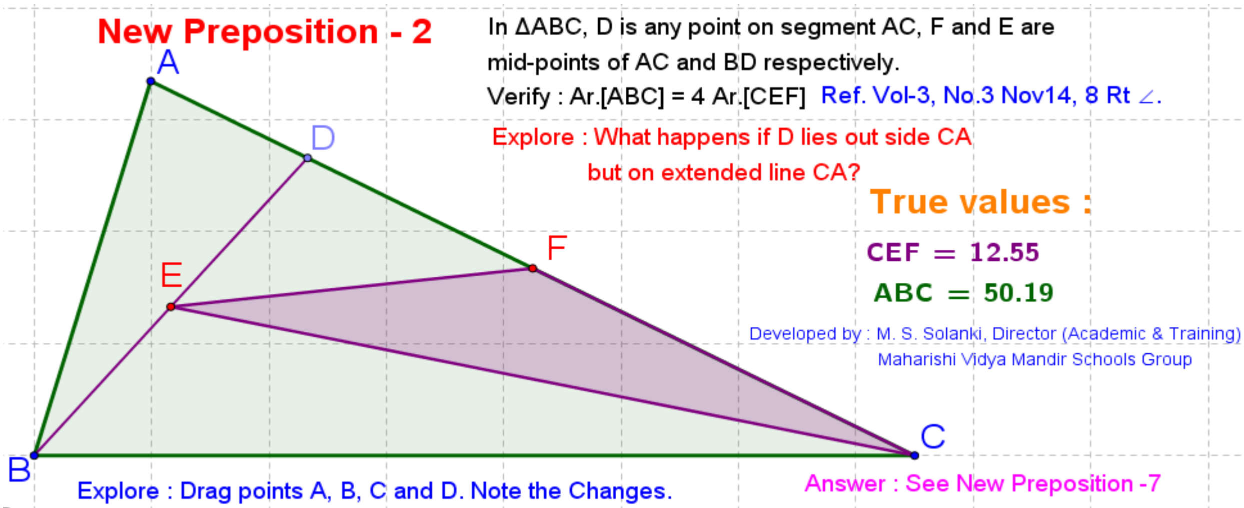
5. In ABC AD BC, BE CA, CF AB, DP AB, DQ BE, DR CF and

   DS AC. Verify points P, Q, R and S (as shown in figure) are concurrent.

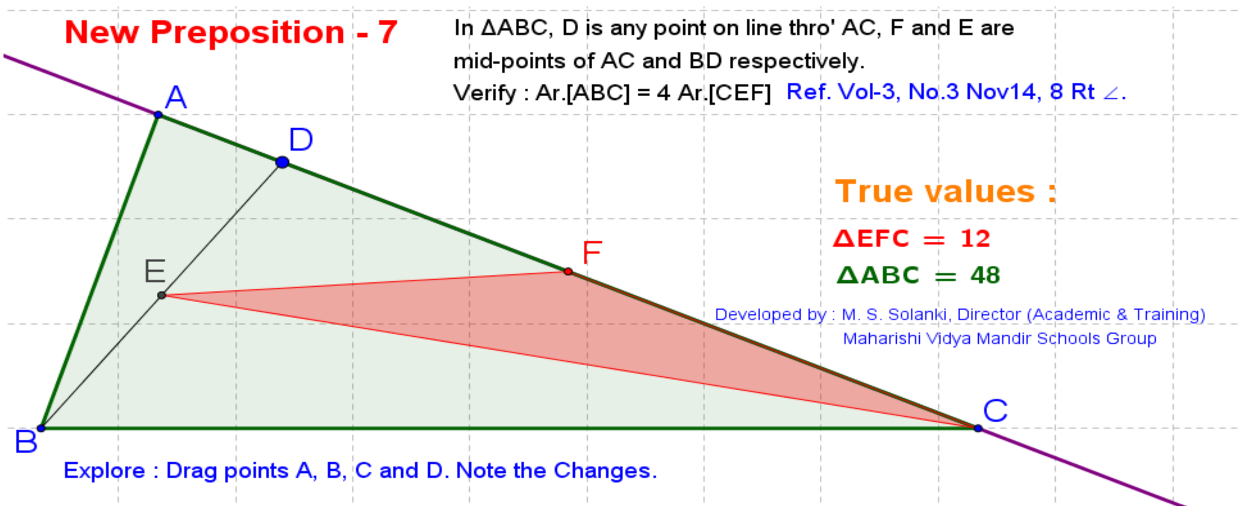
Preposition – 1



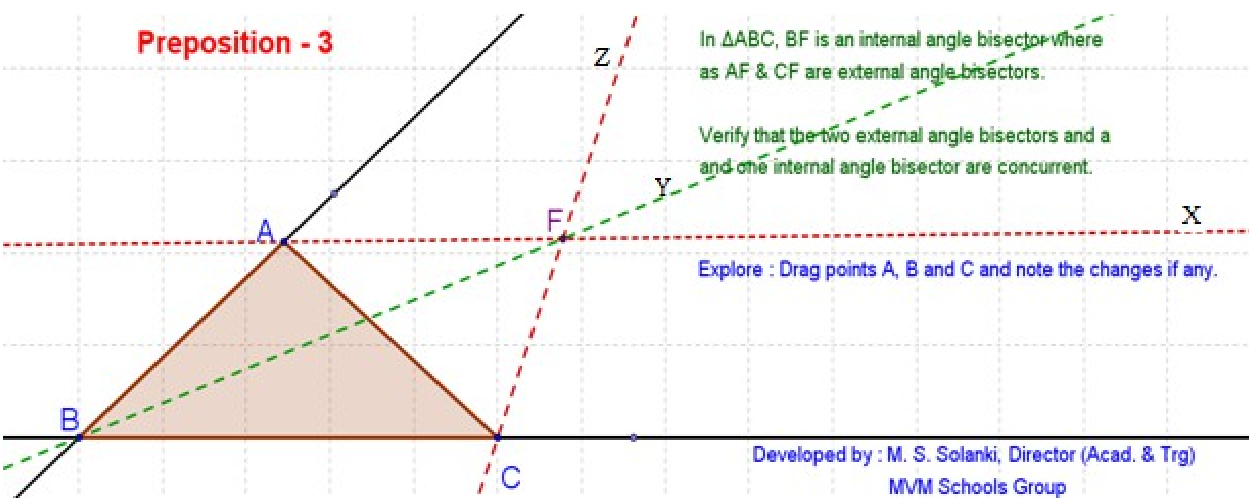
Preposition – 2



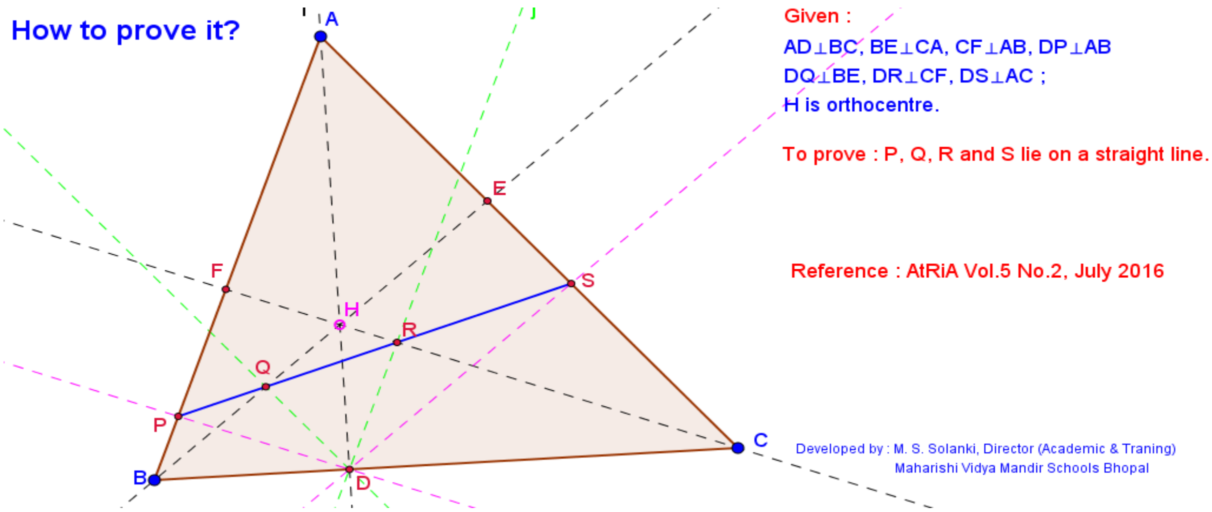
Preposition – 3



Preposition - 4



Preposition – 5



Conclusion :

Thus we see with the help of GeoGebra we can explain the geometric problem/theorem very easily and students can learn by play way method at anytime, anywhere and he can revise them any number of times.

It also helps them to visualize the concept of geometry very well. The effect of visuals remains longer time on the tender minds of the students than the usual chalk and talk method.

Mathematics teachers can use these Non routine applets (beyond the books) to teach the geometry in secondary classes.

The students can use these applets on smart mobile phones or tablets.

This software will also help the students to develop their own creativity.

Reference:

#1 Sanskrit shloka is taken from Vedang Jyotisa shloka #4. It is also given "Indian History of Mathematical Astronomy" by Dr. Balchandra Rao which is published by Gyandeep Publisher, Bangalore.

#2 Most of these prepositions are take from "At Right Angle (=AtRiA)" Magazine published by Azim Premji University, Bangalore.

MIDDLE SCHOOL MATH

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"*Awake, arise and educate, Smash traditions – Liberate*."
Savitribai Phule

In 21st century, the teaching profession has become very challenging. The ease of accessibility to knowledge that is freely available literally at the click of the mouse instead of making the situation easy for the educators has actually made it tough. Greater accessibility does not necessarily really translate into better learning outcome as students do not have the where withal to optimally utilize the info/ piece of knowledge readily available! Here comes the challenging role of the teacher who not only needs to keep her expertise/skills in her subject domain updated but also has to keep abreast of the changing trends in terms of how students learn and additionally help them navigate the complex maze of knowledge that has to be pieced together only under the able guidance of a seasoned educator. It is clear that the teacher's role must be redefined to meet the needs and demands of today's times.

This over load of information also means that the teacher now has to compete for the students' attention.

Then how to motivate the present day tech- savvy *netizens* (read the present generation of learners) towards learning subject like Math?

Let us begin with some fundamental questions:

What makes Mathematics as a discipline special?

Mastering logical reasoning and ability has to happen in a progressive manner. The knowledge of mathematical concepts consolidated at the elementary level gives the strong foundation upon which logical progression is possible at the next level.

Now the teacher's job is to make the students to remember all concepts. How should one proceed now?

The objective can be accomplished if students are attentive in the class. To make students attentive the teaching – learning process should be made interesting. If the teacher can create a positive class room environment, and make learning relevant and engaging, then student can relish the feeling of accomplishment and success.

To create and then sustain the interest level, the teacher can use the play way method of learning.Though it may appear that the students are just playing, they are actually learning valuable skills, including important social skills and cooperation with others.

"Everybody is a genius, but if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid " - Albert Einstein

In a class one can see different types of learners (i.e.) Visual, Auditory and Kinaesthetic learners. Can conventional teaching method reach every single student in a heterogeneous class?

The answer is a big NO. Conventional teaching methods cannot engage everyone to internalize mathematical concepts. So, different innovative techniques are to be adopted for teaching conceptual subject like Maths. One of the many documented methods that have proven effective time and again is the play-way method. Invariably every child likes to play games. It gives free reign to a child’s curiosity and helps the child to enhance their learning ability. The informal framework within with children discover and learn help them to overcome Math-phobia.

Purposeful play experiences can foster deeper learning experiences that a child will remember and internalize. This helps the students to work harmoniously and learn with other students in the class.

Given below are some Mathematics games which can be employed to teach different concepts:

MATH- RELAY: (Any topic other than Practical Geometry and graph)

1. Class can be divided into 4 groups,each group to have one leader (preferably the top scorers who can help others to solve problems).
2. The beginner of the game would get one question to solve; the next person wouldhave 2 questions to solve including the first question which was originally given to the beginner. The third person would have 3 questions. Like this the process continues.
3. After solving the question,the first student will run to second student . Both now join forces to solve/ check the answers, then run to third student. This relay thus continues.
4. Points for the result: The group which completes the relay first will get bonus points, discipline throughout the conduct of the activitymakes up for some more points. The group which gets maximum correct answers will get the certainpoints.
5. The group which gets the maximum points will be adjudged as winners.

MATH- DARGE BALL (Any topic which has more one word questions)

1. The class can be divided into 2 groups. Each group would get a different topic.
2. Instruct the children to prepare one word questions (Max.25).The questions should not be discussed with other groups.
3. One group to form a circle (Group A). The second group(B) would take position inside the circle formed by students of Group A.One student from A will throw the ball to any one student of B(The ball is to be aimed below the knee of the students).If the ball touches the student he/she will be considered out of the game.
4. One question from (A) will be posed to a particular student. If the child is able to answer that, he/she can continue the game and group B will get points. If the chosen students fails to answer, then he/she will be out of the game . The other group, A will get points. This game could continue up to 30 minutes.
5. Next Group B is to form the circle, the other group A will be inside the circle. The game proceeds as detailed above.
6. The top scoring team will be considered as the winner.

SNAKE AND LADDER: (Preferable topic -Playing with numbers / Integers)

Snake and Ladder chart and one word questions to be prepared (well in advance) by the teacher.

1. Divide the class into 2 groups (each group to have one leader).
2. One student from each group to be selected for playing the game.
3. Students asked to role the Die, according to the number obtained, movement of the object (Coin) on the board to be done.
4. If they meet snake or ladder, one question to be asked. If they answered correctly they can use ladder otherwise (wrong answer) they should use snake.
5. Likewise every students will get chance to answer the questions.
6. The team who reaches the destination first will be the winner.

DEAL OR NO DEAL: (Challenging game)

One word questions (25) to be prepared by the teacher. Make a copy available.

1. Divide the class in to two groups. The question paper to be given to them to discuss the answers (2 minutes time can be given).
2. One group will start the game with challenging (they can able to answer 5 questions correctly). The other group will challenge with 6 questions. The process will be continued.
3. Finally one group (consider A) will decide to answer maximum (15 to 20) questions, the other group will challenge.
4. If group (A) answered correctly all the questions (Challenged), they are the winners otherwise group B will be the winner of the game.
   Rules have to be set for the game. Prior planning is required to conduct this kind of game.

THE PURPOSE/BENEFIT OF CONDUCTING MATH GAMES:

All the above games can be effortlessly conducted for middle school level with amazing results guaranteed.

These types of activities provide a comfort level optimal for learning by the students. Rules can be set by the teacher for the participants in such a way that it will accomplish their expectations from students like ensuring the required work CW/ HW is done, given assignments are completed and students are regular to school.

Students will be motivated by their peer group. All the group members will work together to solve problems, each one will help others. It will give chance to the students to come out of Math phobia. The concept taught/revised in this way will remain etched in the mind for long period of time which is the main advantage of conducting games in Math.

In our education system, the percentage of practical studies compared to theoretical is very minimal. Maths is not just a subject to be learnt at school but its real life application is something that makes it an important discipline to master. If you ask children to give the approximate height of a building in metre, most of them would fail to answer. This is so because they can't put their theoretical knowledge to practical use. Great effort is needed especially for a subject like Math to inject more practical oriented teaching than just insist upon memorizing theory.

When teaching area and perimeter to students of standard IV and V, we have to assign students to find the area and perimeter of their own class room, windows, doors and tables. Through such simple activities we can connect math with real life application.

“The best teacher is not the one who knows most, but the one who is most capable of reducing knowledge to that simple compound of the obvious and wonderful”-H.L.Mencken

While teaching we should pose appropriate questions which will provoke and ignite student’s intuition and thinking. Everything is possible if we are a little more creative and enthusiastic towards teaching Maths.

“Education is not the learning of facts, but the training of the mind to think”

Keeping this in mind we all can revolutionize our methods to teaching Maths to young minds.

मुख्य विषय-.   classroom experients and   experiences

वर्गातील गणित अध्यापनातील विविध प्रयोग.

नितिन विनायक पाटिल

वेलीमाता आश्रमशाळा वेली

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विद्यार्थ्यांना आपन गणित शिकत आहो व शिक्षक आपल्याला गणित शिकवित आहेत असे वाटण्याऐवजी आपन खेळत आहोत असे मुलाना वाटले पाहिजे.

अगोदरच म्हटल्याप्रमाणे आमची शाळा ही आदिवासी मुलामुलीं आश्रम शाळा असल्यामुळे पहिल्य वर्गात दाखल होणारी मुले ह अंकज्ञान व अक्षरज्ञान यापासून अनभिज्ञ असतात. म्हणुन सुरवात ही अगदी गणन पूर्व संकल्पनेपासून होते. म्हणुन गणन पूर्व तयारी पासून गणन, अंक वाचन, लेखन, अंकांचा लहान मोठेपणा, मागचा पुढचा अंक, अंकांचा चाहता उतरता क्रम, बेरीज वजाबाकी, गुणाकार भागाकार या सर्व गोष्टी खेळातून तसेच विविध प्रकारचे साहित्य वापरून शिकविण्याचा प्रयत्न आहे.

* गणन पूर्व .तयारी   -   खेळातून व कृतीतून

गणन पूर्व तयारी करून घेतांना वर्गात काही कृती व खेळ घेतले जातात.

* खड्यांचे सारखे ढीग करा. एकत्र करा पुन्हा सारखे ढीग करा

*विविध वस्तू मधून सारख्या रंगांच्या वस्तू वेगळ्या करा, सारख्या आकाराच्या वस्तु वेगळया करा किंवा शोधा. यासाठी परिसरातील झाडांची पाने, रंगीत खडू विविध रंगांच्या गोट्या, काड्या, खडे. इत्यादी परिसरातीलच साहित्याचा वापर होतो.

*वर्गीकरण, तुलना - यासाठी काड्या, खोके, मणी., गोट्या पाने फुले यांचा वापर करून वस्तूच्या आकारा वरून लहानमोठपणा ठरविणे रंग आकारावरुन वेगळे करणे.

* गणन बेरीज वजाबाकी -

गोट्या, काड्या, मणी, खडे, यांचा वापर करून 1 ते 9 पर्यंत गणन करणे व वस्तूंच्या संख्ये प्रमाणे अंकांचे लेखन करणे, वस्तू दाखवुन मोजून मग अंकात त्याचे लेखन कसे करतात ते शिकणे. वस्तू मोजताना काहीच नाही म्हणजे शून्य.

विविध रंगांची मण्यांच्या माळेच्या व काड्या यांचा वापर 1 ते 100 अंक मोजण्यासाठी दशक एकक शिकविण्यासाठी होतो.

* विविध खेळ

संख्या दृढ करण्यासाठी 1) वर्गात जमिनीवर,फरशीवर रांगेने अंक लिहावेत व त्या अंका समोर तितक्या वस्तू ठेवाव्या. 2)वर्गातील मुलांचे दोन गट करावेत, गटातील प्रत्येक मुलाला 1 ते 9 असे क्रमांक द्यावेत, मध्यभागी रेषा आखून रेषेपासून समान अंतरावर दोन्ही संघांना उभे करा, मध्य रेषेवर एखादी वस्तू ठेवावी शिक्षकांनी कोणताही एक अंक सांगावा दोन्ही संघातील त्या क्रमांकाच्या मुलांनी मध्य रेषेवर असलेली वस्तू पटकन उचलावी. 3) मावळ्यानो पळा पळा - मुलांना गोलाकार उभे करून हळुवार पळावे शिक्षक मावळ्यांनो पळापळ असे म्हणतील मुले पळतील शिक्षक मध्येच म्हणतील तानाजी म्हणतात 5 तेव्हा मुले 5 - 5 च्या गटात उभे

राहतील. असे वेगवेगळे अंक सांगून खेळ घेतील.

तसेच संख्यांचा चढता उतरता क्रम लगोरी संचाचा वापर करून दृढ करणे. लगोरी संचातील लगोरी वर क्रमाने लहान कडून मोठ्या कडे संख्या आकार नुसार चिकटून रचण्यास सांगावं सर्वात लहान संख्या वर तर सर्वात मोठी लगोरी व संख्या सर्वात खाली.

* पाढे. - पाढा म्हणजे त्या संख्येची पुन्हा पुन्हा बेरीज होय. तीच संख्या पुन्हा पुन्हा मिळवुन पाढा सोप्या पद्धतीने तयार करता येतो. मुले स्वतः पाढा तयार करू शकतात. तसेच तयार केलेले पाढे पाठ व्हावेत यासाठी खेळामध्ये पाढ्यांचा वापर करण्यात येतो. जसे कबड्डी खेळतांना दोन्ही संघातील मुलांनी चढाईला जातांना कबड्डी कबड्डी म्हणण्या ऐवजी आपल्याला मिळालेल्या क्रमांकाचा पाढा म्हणावा. खेळ प्रत्येक मुलाला आवडतो त्यामुळे मुले आवडीने पाढे पाठांतर करतात तसेच खेळतांना प्रत्येक मुलाने म्हटलेला पाढा वारंवार ऐकतात, दुसरे म्हणजे पाढा चुकल्यावर खेळाडू बाद होईल पाढा म्हणताना पाढा चूक की बरोबर हे ही लक्षपूर्वक ऐकतात.

* गट पद्धतीने अध्ययन व अध्यापन तसेच अध्ययन कार्डचा वापर. -

शालेय सत्राच्या शेवटचा एक तास आम्ही मुलांना गटाने अध्ययनास बसवतो. गटात बसल्यावर मुलांना स्वयंअध्ययनासाठी अध्ययन कार्ड व इतर गटाने कामे देतो. गट बनविण्यासाठी सुद्धा विविध खेळ होतात. जसे - वर्गातील विद्यार्थ्यांच्या संख्येनुसार 5 समान गट केले. गट क्रमांक 1 ते 5 असे क्रम दिले. नंतर आपल्याकडील प्रश्न कार्ड /चिठ्या मधून विविध गणिती क्रिया करावयास द्याव्या त्यातून ज्याचे उत्तर 1 आले त्याने गट क्रमांक 1 मध्ये ज्याचे उत्तर 2 आले त्याने 2 तर ज्याचे उत्तर 3 आले त्याने गट 3 मध्ये बसावे याप्रमाणे गट पूर्ण होतात.

गटात, शिक्षक मुलांना आपण आपापल्या वर्गातील मुलांच्या गरजेनुसार तयार केलेले अध्ययन कार्ड वापरण्यास देतात. त्यात वर्गाच्या पातळी प्रमाणे अंकांचे कार्ड, विविध अंकी संख्यांचे कार्ड, नमुना गणिती क्रिया करून दाखविलेले (बेरीज, वजाबाकी, गुणाकार, भागाकार) डेमो कार्ड, प्रश्नोत्तरे कार्ड अशी विविध प्रकारची कार्ड देण्यात येतात. मुले दिलेली कार्ड स्वतः हाताळतात, गटातील मित्रांसह चर्चा करतात, स्वतः प्रयत्न करून शिकतात.

गटाने अध्ययन करतांना कार्ड शिवाय देखील इतर अनेक कृती, खेळ घेण्यात येतात.

जसे 1 प्रत्येक गटात दोन तीन मुलांना विशिष्ट संख्येत मणी द्या व एकत्र करून बेरीज करायला सांगा.

2 दोघा तिघा कडील मणी कमी जास्त तुलना करायला सांगा.

3.जास्त संख्येतुन कमी वजा करायला सांगा.

4 सर्व मणी एकत्र करून गटात समान वाटणी करायला सांगा.

5.गटातील मणी एकत्र करून वेगवेगळ्या प्रकारे बेरजेच्या रूपाने मांडणी करायला सांगा.

असे विविध प्रकारचे खेळ की ज्याच्यात गणित आहे ते घेऊन गणित शिकणे सोपे व आनंददायी करण्याचा प्रयत्न आहे.

नितिन विनायक पाटिल

P24A00

COMPARATIVE STUDY OF CAI AND CONVENTIONAL METHOD FOR ATTAINMENT AND RETENTION OF MATHEMATICAL CONCEPTS

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1. INTRODUCTION

Mathematics is an indispensable part of informative system. Its practical utility is not understood by many students. They are suffering from Math Phobia. Beside this, teacher cannot cater to the needs of all learners in forty five minutes class. So certain methodology and supporting instructions are needed to enhance Mathematical Teaching Learning Process. Some researchers ([1]-[3]) have found Computer Assisted Instruction (CAI) as effective mode for teaching learning process in Mathematics from primary to higher secondary level. But there is lack of researches on CAI in Mathematics for higher studies in Indian situations. Keeping in mind the problem of better understanding, clarity of Mathematical Concepts and maintaining the quality of pedagogy of Mathematics, the researcher developed Computer Assisted Instructional Material (CAIM) for attainment and retention of Mathematical concepts at undergraduate level ([4] , [5]).

1.1 Operational Definitions of terms used

1.1.1 *Computer Assisted Instruction (CAI)*

In this study, CAI is mode of instruction for providing self-learning material to the undergraduates through computer which is learnt at their own pace. The instructional material for CAI is prepared in the form of frames followed by self -evaluated questions in multiple choice formats.

1.1.2 *Conventional Method (CM)*

In this experimental study, Conventional Method (CM) means that method of instruction where the teacher plays a major role and the lesson is also planned. Objectives are framed and stepwise evaluation is done at every stage. Same material used for CAI is utilized in the planning of CM.

1.1.3 *Undergraduates*

Undergraduates are female students studying Mathematics in B.A./B.Sc. of Banasthali Vidyapith, Rajasthan, India in this comparative study.

1.1.4*Attainment and Retention*

Attainment refers to the scores obtained by undergraduates on Attainment Tests and retention refers to the scores obtained by undergraduates on Retention Tests in the form of Criterion Referenced Tests (CRTs). These tests have been developed by the researcher for CAI and CM on selected subject area of Algebra, Calculus & Numerical Analysis in Mathematics.

1.2 Objective of the study

The main objective of the study was to compare the effectiveness of CAI and CM for attainment and retention of Mathematical Concepts at undergraduate level

1.3 Hypotheses

According to the objective of the study, the null hypotheses were framed as follows:

- H_{01} There is no significant difference between mean scores of undergraduates in experimental group using CAI and control group through CM for Mathematical Concepts at Entry Level.
- H_{02} There is no significant difference between mean attainment scores of undergraduates in experimental group using CAI and control group through CM for attainment of Mathematical Concepts.
- H_{03} There is no significant difference between mean retention scores of undergraduates in experimental group using CAI and control group through CM for retention of Mathematical Concepts.

2. METHOD AND PROCEDURE OF EXPERIMENT

2.1 Design of the study

In this study, pre-test - post-test equivalent - groups research design was used [6, p181].

2.2 Validation of Tools for the Data Collection

The researcher herself developed CAIM. It was developed in Microsoft Office Power Point 2007 inserting hyperlinks and actions. It included Introduction to Matrices; Types of Matrices; Basic Operations on Matrices; Elementary Matrix Operations and Echelon Form of a Matrix, Formal definitions of the limit of a function; Continuous functions and Classifications of discontinuity in Limit and Continuity, Introduction to Errors in Numerical Computations: Bisection Method; Regula Falsi Method; Newton Raphson Method; Order of convergence for Numerical Methods in Solving Algebraic and Transcendental Equations. CAIM resembled with a test but it was not a test (Figure 1). It was self-learning material. The criteria of 85 % success were adapted to evaluate the efficiency of CAIM. Validity and suitability of these tools was calculated in terms of gain score for CAIM in pilot study of 33 undergraduates. For this experimental study, an Attainment Test MCT (Mathematical Concepts Test) was prepared by adding all test items to measure the attainment of Mathematical concepts. Parallel Attainment Test PMCT was also prepared on the same lines. The items in MCT and PMCT were of various types such as Objective Type, Completion and Short Answer Type. The content validity of the MCT, PMCT and Scoring keys was determined by the subject experts. Individual and small group try-out was carried for the editing of content material. Reliability co-efficient of measurement tools was Kappa (.89) in terms of the consistency of decision - making process across as alternate forms of attainment test at mastery level. It was calculated in terms of mastered and non-mastered group for pilot study [7].

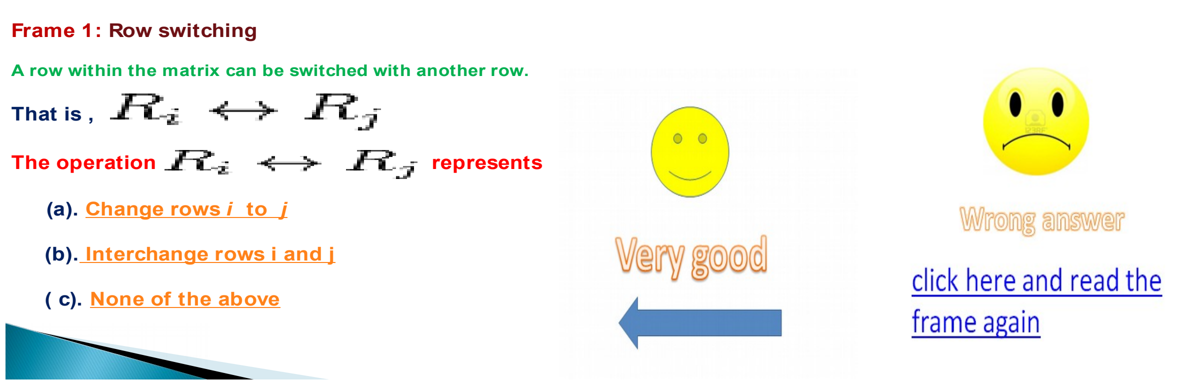

Figure1: Sample of Frames for CAIM

2.3 Procedure of final experiment

The sample for this experimental study was selected from Banasthali University, Rajasthan, India having smart computer lab. Purposive sampling technique was used to select groups by the researchers. Method and Procedure of Experiment is depicted as follows (Figure 2): In the beginning, 100 under graduates (females), who offered Mathematics as elective subject, were selected. Raven's intelligence test was given to students for matching their level of intelligence. Both groups were also pre-tested with Mathematical Concept Test (MCT) at entry level to find out whether they have not learnt the unit beforehand. On the basis of the scores obtained in I.Q. Test and Pre-test, the undergraduates were matched and equally distributed in experimental and control group for final experiment. After controlling the effects of confounding Variables, the sample size was reduced to 90 at the time of analysis. Each group had 45 students. The experimental group was exposed to CAI and control group was instructed through Conventional Method. After experiment, MCT was conducted (at Attainment Level) and PMCT was conducted after seven days (at Retention level).Method and procedure of final experiment is depicted in Table 1.

Table1: Method and Procedure of Experiment

3.

Groups	Pre stage (Entry Level)	Treatment (Matched Group)	Post 1 stage (Attainment Level)	Post 2 stage (Retention Level)
Random	Intelligence Test, Attainment Tests (Pre-test)	CAI (Experimental Group)	Attainment Tests (Post-test I)	Retention Tests (Post-test 2)
Random	Intelligence Test, Attainment Tests (Pre-test)	Conventional Method (Control Group)	Attainment Tests (Post-test I)	Retention Tests (Post-test 2)

RESULTS AND DISCUSSIONS

The scores obtained in Pre-test, Post-test1 & Post-test2 were recorded for data analysis In order to analyze the data, SPSS v.16 was used. The effect of instructional material for CAI and Conventional Method in attaining and retaining Mathematical Concepts between the groups was evaluated. Gain Ratio in terms of learning gains were also calculated for both modes of instruction (Table2). Independent t-test was used to compare experimental group and control group at Pre stage, Post I stage and Post II stage (Table3). Table2 indicated that the learning gain for experimental group using CAI was 88% where as 83% for CM in Control group for attainment.

Table2: Comparison of Learning Gains

Sr. No.	Level	Mode	Total Average Gain Sores	Learning Gain for Entire Group on Average
1	Attainment	CAI	.88	88%
		CM	.83	83%
2	Retention	CAI	.85	85%
		CM	.78	78%

It is also clear from the findings that learning gain for experimental group using CAI was 85% where as 78% for Conventional Method for retention of Mathematical Concepts. This means that instructional material for both modes of instruction was satisfactory at both the level of learning. Based on these results, material was taken as good enough for use during experimental study.

Table3: Results of Independent t-test

	Group Statistics				Independent Sample Test		
					t-test for equality of mean		
Stage	Mode	N	Mean	S.D	T	d. f.	Sig.(2-tailed)
Pre	CAI	45	57.24	19.005	.054	88	.957
	CM	45	57.02	19.841			
Post I	CAI	45	228.91	12.938	3.837	88	.000
	CM	45	218.84	11.928			
Post II	CAI	45	222.33	12.424	5.358	88	.000
	CM	45	209.04	11.066			

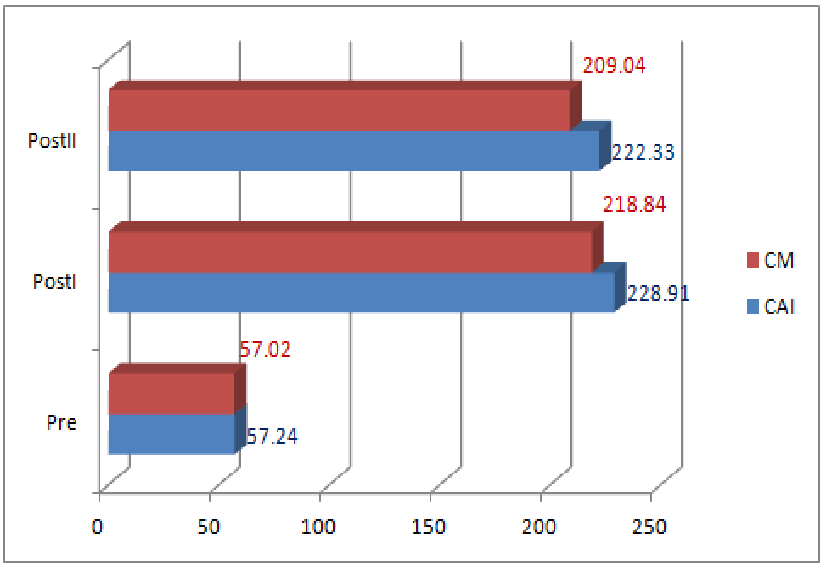

Figure2: Graphical Representation of Comparative Results

Findings (Table3 and Figure2) pointed out that entry level behavior of undergraduates for both the groups were similar. The reason might be that undergraduates had not read the particular Mathematical Concepts before the treatments. The results also indicated the significant difference (.000) between the mean attainment and retention scores of experimental group and control group at (Post I and Post II stage). Therefore null hypothesis were rejected. Finding supported the effective influence of CAI for attainment and retention of Mathematical Concepts. On the basis of these discussions, it may be resolved that CAI being an auto instructional mode of instruction helped in improving the attainment and retention of Mathematical Concepts at undergraduate Level. CAI offered a valuable means for improving mathematical knowledge at undergraduate level. Administrators should motivate faculty members to develop CAIM for educating mathematical concepts in higher education.

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A Study to Check the Effectiveness of Innovative Mathematical Resources on Numeracy among Underprivileged Children

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Abstract

Introduction

The Right To Education (RTE) acts entitles each and every child from age 6 to 14 free and compulsory Education. But the quality of education for the underprivileged children in various child care homes and government schools around different parts of the country is still a serious concern. One of the most immediate aim of education is to make each and every child capable of employment and provide them necessary skills to live their life easily.

There is a huge focus on the term 'Literacy' but there is an equal need for the 'Numeracy'. Numeracy provides base to every child for basic mathematical skills which can be very helpful in their finances and money management. Numeracy can open so many ways for young population in the widely spread job market of globalized economy. The mathematics has always been a fearful subject for most of the students. The traditional and same blackboard teaching methods don't excite students. There is a need for some innovation in the normal classroom to make mathematics interesting for them which could help them in attaining basic Numeracy skills.

The Study

The present work is based a small scale research done with underprivileged children from child care home of Haridwar to look for the effectiveness of the classroom teaching when it is equipped with innovative teaching-learning resource for mathematical learning. Innovative Mathematics teaching-learning resources are defined as various need based, localized, age appropriate manipulative, and instruments which are child friendly and facilitate learning. The study was conducted on a sample of 30 boys of Bal Kunj situated in Haridwar district of Uttarakhand. Two concept namely Arithmetical Operations (BODMAS) and Basic Shapes (Squares & Rectangles), fundamental to Numeracy were chosen which could be helpful in daily life Mathematics.

The data was collected through pretest-posttest research design. For pretest, the data was collected in two phases. Firstly, the informal discussion was done with the students regarding the number sense and arithmetic operations. Secondly, formal testing was done on concept based and application based questions. The application based questions were posed immediately after the same concept based questions

to give a little clue to the children. This was also done to prevent them from any anxiety during the test. Before starting with these tests, an informal discussion was done with the group of students who were to be included in the sample collection. They were informed about the assessment. After going through with the results of pretest, specific problem areas were identified and specific tools or resources were developed by the researcher under the guidance of the research supervisor. The tools consisted of two self – developed mathematical games and third tool was based on tangram. Two intervention plans were made after analyzing the results of the pretest. Those two plans were made to focus on concepts namely, Arithmetical Operations (BODMAS) and Basic Shapes (Squares & Rectangles). Each intervention plan was of five hour duration. The plan started with exploration with the concepts using mathematical tools designed for that concept. Children were encouraged to discuss, argue, play around, explain and explore through put the session. The tools were handed over to the child care home teachers to allow children to use/play it after formal school hours.

The first game, titled, *Conquer the Empire, was* a board game based upon the concept of integers and arithmetic operations. The games provided visualization counting with integers. Also, it also helped them in making game planning and strategies.

The second game titled, *BODMAS Run* was based upon various one digit BODMAS operations. It aimed to performing and strengthening the BODMAS operations with fun.

The third tool was based on existing popular game, *Tangram.* Different physical stencils were used of the tangram pieces to explore the concepts of basic shapes, patterns, perimeter and area.

The concepts of numeracy that were included in the research are arithmetic operations (BODMAS) and patterns made by squares and rectangles along with their area and perimeter. The selection of these concepts were made initially in broader context. The number sense and basic operations with patterns were included in the pretest to look for the specific areas where there is a need to intervene and help them. After analyzing the results of pretest, it was discovered that most of the students were good in number sense. So, the specific focus was made on the other concepts as the time given to intervene in their normal school routine was very less.

After the intervention, posttest was conducted on the same concepts though the structure and organization of question items was made bit difficult. Detailed comparative analysis of data collected through pretest and posttest was done. There was a significant improvement in their posttest performance, classroom participation and level of interest. They were highly interested in exploring the tools by themselves and learn something new. The major change was observed in their classroom behavior. They were keen to play and this keenness let them learn with fun.

Conclusions

The findings of the study suggested that hands on tools was really beneficial for the underprivileged children in learning Mathematics. It helped them to visualize the concepts and hence foster their understanding. They found activity oriented classroom more interesting. They were already underprivileged in terms of their social, economic, emotional environment. So, it naturally became difficult for them to be focused on learning, specially, Mathematics, which did not make much sense to them in normal blackboard teaching. Use of activity oriented innovative tools helped the researcher to grab the attention of students and to make a shift from normal blackboard teaching to interactive teaching environment.

The study suggested that innovative mathematical tools can also be very useful in building up many of the concepts in the limited time frame. It is also important to introduce such tools along with classroom teaching. They solely cannot play such role.

The study provided helpful insights in solving many issues regarding the education of underprivileged children. The classroom environment also plays a significant role in better learning of the children. The study can be replicated on large scale to validate the findings and the results obtained could be used as premise for the introduction of innovative mathematical tools in the classroom for teaching. Such innovations in a classroom could help underprivileged children in getting the basic skill of Numeracy that will help them in grabbing the good jobs in the competitive market of globalization.

Acknowledgement

I would like to thank **Cluster Innovation Centre (University of Delhi)** and my mentor **Dr. Jyoti Sharma** for constantly providing me support and guidance whenever it was needed. Her generous advice and cooperation is largely appreciated. I am highly grateful to **Mrs. Usha** for permitting me to conduct the study in Bal Kunj. Her cooperation and help was very helpful in carrying out the study smoothly.

Keywords: Underprivileged, Numeracy, Innovative Mathematical Tools, Mathematical board Games

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The State of Undergraduate Mathematics: A Student's Perspective

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Undergraduate mathematics is one of the important aspects in higher education, but less talked among educators and academicians. Venkatraman, Sholapurkar, & Sarma (2012) in their article present a critical overview of mathematics education at tertiary level in India. The article covers historical developments of curriculum to current teaching and training processes in tertiary mathematics. Within these aspects of curriculum and teaching processes, the author tries to present some aspects of undergraduate mathematics scenario in the form of his views and experiences as a student of mathematics at university level. The author has been a student of mathematics (honours) at one of the colleges of one of the largest universities in Bihar and after that he moved to a university in Delhi to pursue post-graduation. As a part of studying mathematics in two different universities and being involved in the field of mathematics education, the author puts down his views and experiences mostly based on poor status of tertiary mathematics.

During his undergraduate course in Bihar, the syllabus for mathematics (honours) course of the university in Bihar was not much different from other universities across India as it comprised core papers like Algebra, Calculus and Analysis. However, it did lack something as students were not given opportunity to study Probability and Statistics. 'Probability and Statistics', Numerical Analysis, Spherical Trigonometry are mentioned as optional papers in the syllabus. Unfortunately, the students didn't get a wide choice to study. Perhaps, 'Probability and Statistics' is never taught across any college of the university. It was later that the author came to realise that he didn't get taught one of the great tools which every mathematics graduate should know -'Probability and Statistics'. Instead of 'Probability and Statistics', Spherical Trigonometry was taught. Surprisingly, the department had neither any observatory nor any practical approach for this paper. The author doesn't have complaints on Spherical Trigonometry being taught but what about 'Probability and Statistics'? Moreover, the university admits a lot of students who are more oriented towards preparing for banking services and government examinations. Wouldn't teaching 'Probability and Statistics' serve a better purpose? Analysing such scenario, the author wonders if there is any board/body which looks after such variations and sensitivity of mathematics curriculum at regional level. The students didn't get exposure to some of the classic books at the undergraduate level. There are some textbooks authored by local professors (mainly of Bihar (Patna) region) which are quite popular in the market. Unfortunately, the students didn't have much choice of textbooks as the market is flooded with only these books. It might be because the university professors recommend only these books. These textbooks appear nothing more than a book full of notes. The case worsens as students are not even aware of some classic textbooks like *Mathematical Analysis by Rudin*. Had the teachers made students aware about such books, students would have tried to access such books. But, how can a student access these books when he/she doesn't even know about the existence or availability of such work? There is also a lot of talk on the history of Indian mathematics. The author as a student also experienced a lack of inclusion of such work in college textbooks. There are various Indian contributions relevant to undergraduate mathematics and students would feel more connected and interested in learning after coming across Indian names. Hardly, the students came across any Indian name in textbooks including 'Spherical Trigonometry'. Students' attendance often remains low in the department and as a result it doesn't favour organisation of various training programmes in the department. Most of the mathematics students joining such colleges and universities in Bihar come from rural background and from a system of schooling where mathematics education is in poor state. Such students require special teaching from professors and the poor state of undergraduate mathematics worsens the situation. Many of them loose their interest in mathematics by the second year of graduation and passing examinations becomes their sole goal instead of mathematical learning. They also suffer from language transition (Hindi to English) as majority of them have studied school mathematics in Hindi. The author did a small scale research related to language transition among mathematics (honours) undergraduates of Delhi University. These undergraduate students had studied mathematics at school level in Hindi medium. Students did express their problems due to language transition and how it affected their mathematics learning at university level (Mishra & Sharma, 2017). Carrying on this work, the author did a survey among students pursuing post-graduation in Mathematics at Delhi University. At post-graduation level, there were only 1.4% students who had studied mathematics at graduation in Hindi medium and 11% students were those who had studied mathematics in non-English medium at school level. Does it reflect poor entry of non-English background students into higher mathematics? Assessment process also lacks at the undergraduate level. Unfortunately, the markets provide a readymade solution to everything from examinations to project works. For students in Bihar, the *Guess Book* or the *Guide Book* proves more than handy to them as the questions asked in university annual examinations are the same as published in the *Guess Book.* At the post-graduation level of mathematics in Bihar, projects usually get managed with the copy and paste approach and the current boom in internet usage has eased such process of submitting plagiarised materials. When it comes to viva examination at PG level there, students are often asked questions related to school level mathematics. There is also a lack of teachers related to special papers in mathematics departments of various colleges in Bihar. UGC recommends programming courses, which involve use of Information Technology, but in these conditions, such recommendations are far from reality in such universities in Bihar. Computer rooms remain locked throughout the year. Most of these experiences are related to a university in Bihar but these instances don't mean that everything would be in good state in reputed universities across India. The author often came across students of a university in Delhi where they were not aware of various aspects and relevance of their papers. For example: The author was involved in a project on mathematics and music. During his work, he met a PhD student in mathematics. She was shocked to know about the project and asked, "How can mathematics be in music"? Apart from these experiences, challenges faced by teachers in university mathematics departments and other issues such as unawareness among mathematics students of Bihar about programmes like MTTS, Madhava competition, scholarships etc. also remain a concern. For instance, the post graduate mathematics department in the author's university in Bihar got new lecturers in 2017. In an interaction with the author, the newly appointed teachers expressed their concerns as they face problem in teaching because students at PG level are not even well equipped with basics in undergraduate mathematics. Sometimes they (teachers) are puzzled over what to teach.

The author doesn't want to generalise these experiences for the state of tertiary mathematics, but wants to draw attention of policy makers and academicians concerned with higher mathematics by bringing out such instances which are prevalent in college and university level mathematics. If such instances are not dealt properly, it would create problem for the great human resource engaged in studying mathematics as well as for Indian higher education. It is important to review the changes undergoing in curriculum of undergraduate mathematics and activities taking place at ground level, in remote areas. Perhaps, the purpose of higher mathematics especially at college level is not being met. To improve tertiary mathematics and especially the undergraduate mathematics scenario in India, it is important that such aspects are dealt sensitively by institutions and bodies related to higher mathematics.

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Analysis of Student Learning in the Concept of Area and Perimeter in Indian English Medium Private Schools

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Several authors have attributed students' poor performance in the topic of 'Area and perimeter' on a tendency to learn the formulas by rote (Simon & Blume, 1994; Tierney et al., 1990). Researchers have also shed light on Indian students' misconceptions in the concept of area and perimeter (Kanhere, Gupta & Shah, 2013). This paper is based on the analysis of student response data on large-scale assessments in English medium private schools to answer these questions: Are students really understanding the concept of area and perimeter? Or is it just limited to plugging in numbers in a formula and compute the area of a shape and to what extent in general? Do they think that non-standard shapes with curved boundaries don't have the area or perimeter? Is the misconception among students prevalent in grades 5 - 7 when these concepts are exposed to? Is it found predominantly in students of a particular math ability? What could be the possible reasons for the misconception arising? This paper also talks about the role of a good assessment item in eliciting misconceptions if any in the context of the concept of area for effective teaching-learning of the concept. We believe this will inform a better curriculum and its transaction at teachers' end and thereby better learning among students.

Keywords: Area, Perimeter, Eliciting misconceptions, Non-standard shapes, Formula-based learning

The National Curriculum Framework, India (2005) expects the upper primary syllabus to emphasize the development of mathematical understanding and thinking in the child. The emphasis in the syllabus is not on teaching how to use known appropriate algorithms, but on helping the child develop an understanding of mathematics and appreciate the need for and develop different strategies for solving and posing problems. There is also a need to ensure that students are not rushed into working with area formulas, instead being provided with better foundations on the concept of area in their early years of schooling (Booker et al., 1992; Batista, 1982; Latham & Truelove, 1980; Williams & Shuard, 1982; Dickson, 1989; Osborne, 1980). Latham and Truelove (1980), Williams and Shuard (1982), Dickson (1989) and Osborne (1980) all discuss this rush to formulas, and describe the need for better foundations to be made in the early years of schooling. Doig, Cheeseman and Lindsay (1995) stated that 'confusing perimeter with area is a well-known problem with young children', (p. 232), while Reys and others (1984) found that there was often confusion between perimeter and area. They attributed this partly to a lack of basic understanding of an area, and partly due to the premature introduction of formulas.

We investigate Indian students' learning in the said concepts based on the student response data on items that are part of different standardized large-scale assessments like ASSET[1] and Detailed assessment[2] taken by students of English medium private schools. In few cases we use the student response data collected on these questions through an intelligent tutoring system, Mindspark[3].

Eliciting student misconceptions

There are some commonly found misconceptions, that we have come across in the concept of area and perimeter in the Indian students.

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[1] A diagnostic test developed by Educational Initiatives taken by about 3.5 lakh students every year across different classes of English medium private schools in India and few Gulf countries with students of Indian origin

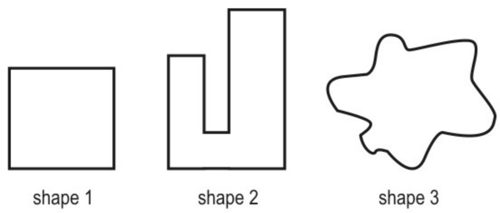
[2] A topic-wise testing tool that aims at providing immediate feedback to teachers on the current level of student understanding in the topics taught and completed in the classroom

[3] Mindspark is an online personalized and adaptive learning platform where students with paid subscriptions by Indian English medium private schools learn along with usual classroom teaching.

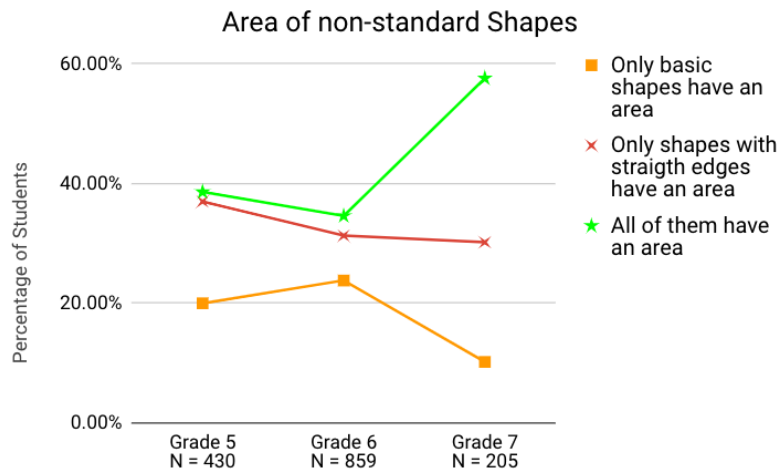
Misconception: A non-standard shape with curved boundary doesn't have an area

Most of the students think that "only a basic (standard) shape like a rectangle, a square or a triangle have an area and a non-standard shape (a shape with a curved boundary) doesn't have an area." Item 1 shown below from the ASSET elicits the misconception. About 64% of 26158 grade 6 students had the misconception. To see the trend of the misconception across grades, we also collected data on the same question through Mindspark. Based on this data, around 50-60% of about 1500 students in grades 5 - 6 think that only shapes with straight edges (polygons) have area.

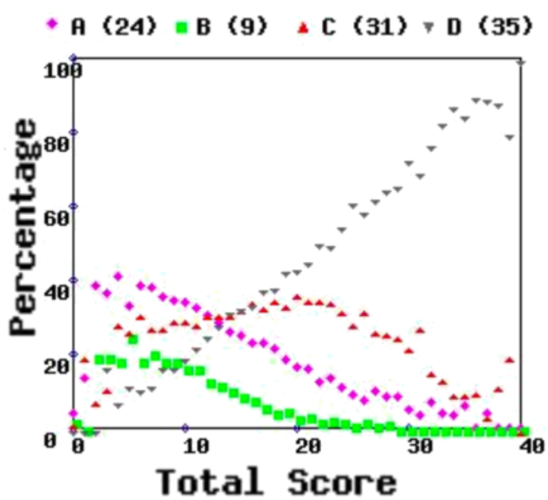
Shown here are 3 shapes drawn on a piece of paper. Which of them has an area?



A. only shape 1
B. only shape 2
C. only shape 1 and shape 2
D. All of them have an area



Source: data collected through Mindspark



Item Response Curve (IRC) of ASSET Data of Grade 6 (N = 26158)

In ASSET, the students selecting [A], 24% of the total, have the misconception that only basic (standard) shapes have an area. The students selecting [C], 31% of the total, have the misconception that only the shapes with straight edges have an area. From the IRC curve, it is evident that the former misconception is most prevalent in student of low math ability (score less than 15, out of a total score of 40) and starts to reduce in medium math-ability student (score in range 20 -30) and thereby almost absent in student of high math ability. Whereas the latter misconception is prevalent in both low and medium math ability students and even exists to some extent in students of high math ability.

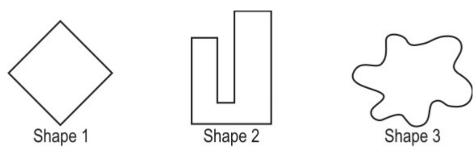
Item 1: Area of non-standard shapes Number of students = 26158

Misconception: A non-standard shape with curved boundary doesn't have perimeter

The concept of perimeter is mostly taught using shapes with straight edges and hence possibly most of the students think that a shape with curved boundary doesn't have the perimeter. Item 2 from the ASSET elicits this misconception. Only 19% of 23823 grade 7 students have understood the concept correctly. Around 51% of the students think that shapes with only straight edges (polygons) have a perimeter. Note that this misconception is more in the case of perimeter compared to the area (area of non-standard shapes is discussed using item 1).

As seen in the IRC, about 80% of the students with total score of 20 or less, out of 40, think that only shapes with straight edges have a perimeter. Even about 20% of students with high math ability (score of 30 or more) have this misconception. About 90% of the students with low math ability (score less than 10) think non-standard shapes with curved boundary don't have a perimeter.

Shown here are 3 shapes.



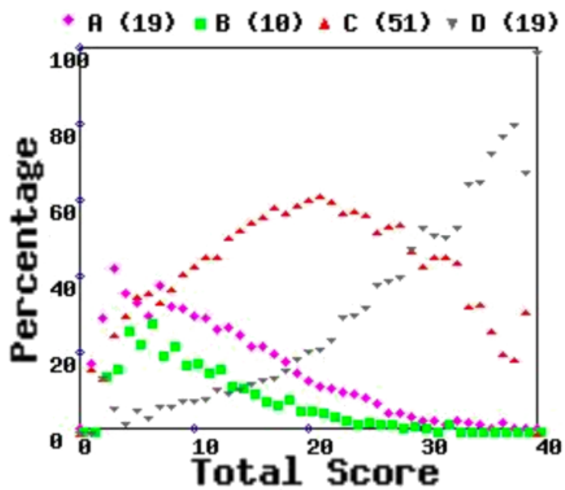
Which of them has a perimeter?

A. shape 1 only (19.3%)

B. shape 2 only (9.7%)

C. shapes 1 and 2 only (51.2%)

D. All of them have a perimeter. (19%)

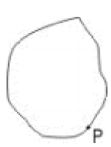


Item 2: Perimeter of non-standard shapes (Item and IRC) Number of students = 23823

Confusion between area and perimeter

Item 3 shown below from the ASSET tests if students understand that perimeter is the length of the boundary of a shape. About 36% of 29252 students of grade 6 answered as 'area of the shape' in the question instead of the perimeter. These students seem to be confused between area and perimeter.

Anu draws the following shape on paper.



She starts drawing from P and ends at the same point as shown below. She does not take the pen off the paper from start to finish nor go back at any time. The length of the line traced by her pen is the

A. area of the shape (~36%)

B. width of the shape

C. height of the shape

D. perimeter of the shape (~47%)

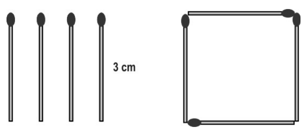
Item 3: Perimeter as the length of the boundary Number of students = 29252

About 25% of 430 students of grade 6 and 7 doing Mindspark shared perimeter is the amount of space enclosed within the boundary of a closed shape when they answered as perimeter for the above question (though with different answer options). Only one of the 4 students in Bengaluru interviewed by one of the authors could justify the answer and say perimeter is the length of the boundary of a shape. We have seen similar patterns on other student interviews in classrooms conducted in the past.

Estimation of an area of a shape is taught in grades 5 and 6 using non-standard shapes and because of this some of the students might be answering as ‘area of the shape’ by merely looking at the non-standard shape.

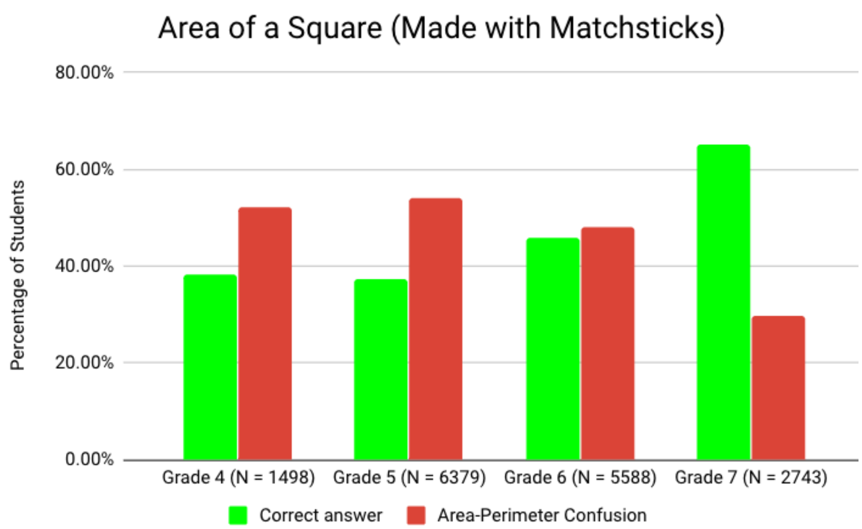
The confusion between area and perimeter is not just limited to the level of understanding the concepts (definitions) but also in applying formulas to compute area or perimeter of basic shapes as evident from the response data of item 4 below. Large number of students from grades 4 - 7 are seen to be answering the area of the square with side 3 cm as 12 (4 × 3 cm) applying the formula of perimeter of a square.

A square is made using 4 matchsticks.
Each matchstick is 3 cm long.



What is the area of the square?
A. 3 sq cm
B. 7 sq cm
C. 9 sq cm
D. 12 sq cm

Source: question in Detailed Assessment



While confusion between area and perimeter goes down moving higher up the grades, it is still substantial (~30%) at grade 7.

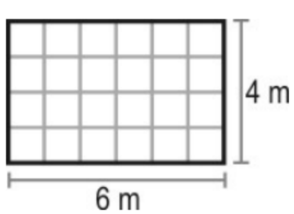
Item 4: Area of a square made with matchsticks

Number of students = 16208

Role of questions in eliciting students’ real understanding

Commonly, to ask for area of a rectangle, 2 side lengths are given in a figure and to ask for perimeter of a rectangle 4 side lengths are given. So when students are asked to find area of a rectangle when all the 4 sides are given in the figure, many of them end up finding perimeter. This also shows how students’ confusion between area and perimeter is better elicited by asking right sets of questions. Student response data on items 5 and 6 below from Detailed Assessment shows this.

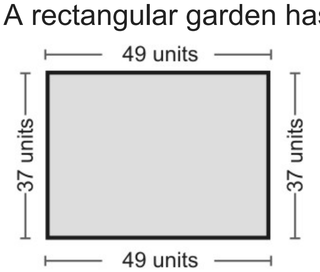
What is the area of the shape shown below?



A. 10 sq m **B.** 20 sq m
C. 24 sq m **D.** 26 sq m

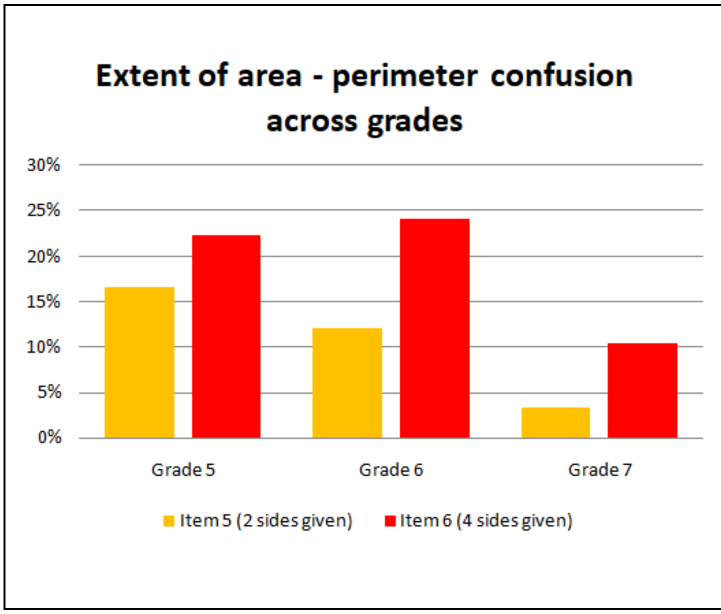
Item 5: Finding area with 2 side lengths given

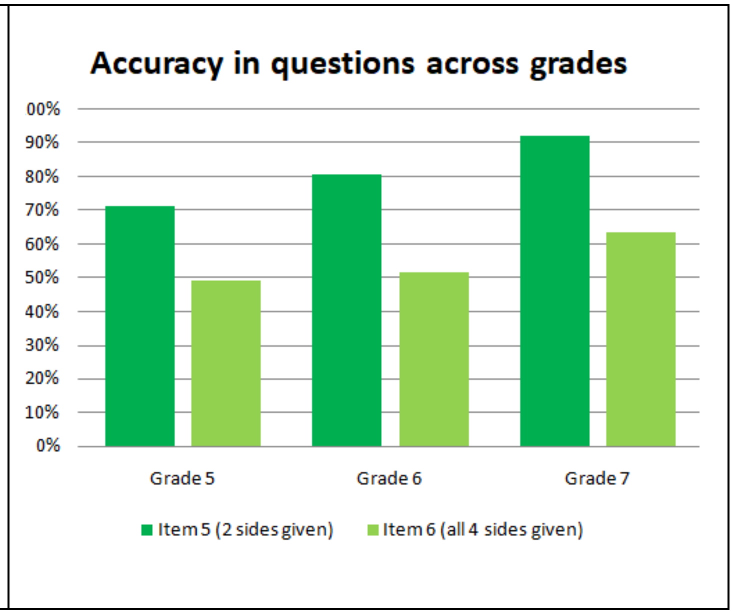
A rectangular garden has sides as shown above.



How will its area be calculated in square units?
A. 49 + 37 + 49 + 37 **B.** 49 × 37 × 49 × 37
C. 49 + 37 **D.** 49 × 37

Item 6: Finding area with all the 4 side lengths given





Discussion and conclusion

Apart from the confusion between area and perimeter of a shape, not understanding how the formulas to find area and perimeter of a rectangle are derived adds to students resorting to rote learning and such errors. The analysis of student response data seem to indicate that even in most of the Indian classrooms the emphasis seems to be more on teaching and applying the formulas without knowing how they work. Exposure to questions of certain type predominantly (computational problems) and not giving adequate exposure to types of questions eliciting conceptual understanding of area and perimeter (like items 1, 2 and 3) possibly leads to not developing the desired understanding of area and perimeter.

Students should be encouraged to calculate the length of the boundary of shapes and real-life objects before introducing the term perimeter. Then they should be encouraged to derive the formulas for area and perimeter of basic shapes. Enough exposure should be given on estimation of area and perimeter of both standard and non-standard shapes before introducing the formulas. Activities like forming different rectangles using a straw on a square grid or geoboard, computing the area and perimeter of each using the grid and tabulating them draws attention to the fact that the perimeter of all the rectangles so formed remains the same and the area varies. Asking students to compute both area and perimeter of a given shape will also help them avoid confusion between the two. We recommend that only after students are clear about the attributes of area and perimeter, a teacher should introduce the formulas to calculate them for various shapes so that they could internalize the concept.

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My experiments with Math teaching

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Introduction

The greatest challenge of being a math teacher is to convincingly change the mindset of the students that math is not everyone's cup of tea. This is the one thing that we should not believe as teachers. Our job simply is to make every student fall in love with Math, however impossible this may sound. Being a math teacher for about a decade now and having taught multiple levels from grade six to grade twelve, my basic tenet thus far has been to engage each and every student in the class.

Background

In an inclusive school like ours, where all kinds of students irrespective of their background are welcome, class room challenges are many. A classroom is a mixed bag of students with specific learning disabilities, first generation learners and students with high aptitude for math Within these categories there are willing learners and unwilling students.

Some of my experiments with math teaching for grade 6 students are summarized below with examples.

Feel mathematical terminology:

Middle school children connect less to the logical aspect of math and mathematical terminologies. For example, when algebra was introduced to the 6th grade students, some of them found it difficult to remember algebraic terminologies like **variable, constant and co-efficient.** Therefore, we formed names like **Mr. Unknown**, for a **variable.** He is the one who has the power to change the answers depending on what values you give him. **Mr. Stubborn** for a **constant** who will retain his value and we do not have the powers to change him.
Co-efficient was called a '**Dear friend of Mr. Unknown'** who co exists with him all the time.
This kind of personifying mathematical terms was a big break through. For a while all the questions had these given names in brackets.

Math is always about being fair

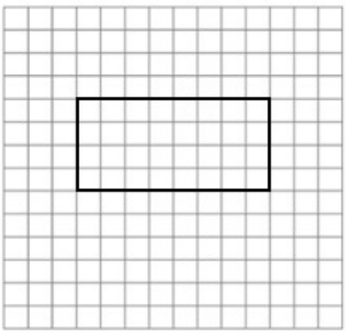
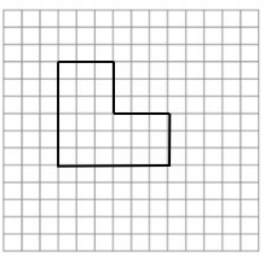
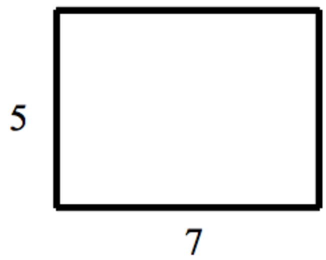
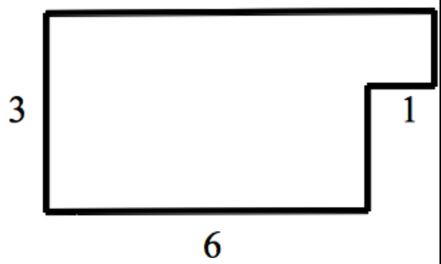
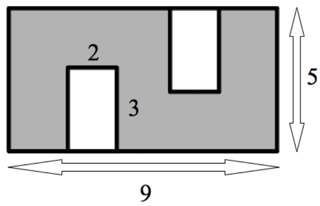
This is the punch line used while adding unlike fractions, balancing the equations etc. The most common mistake committed by students while adding unlike fractions is to add numerators and denominators – they were told that this was simply unacceptable because they were not being fair. Constant reiteration of this would result in correcting their mistakes and internalizing the concept.
ANOTHER application of this idea was with balancing the equations. They were taught that when we alter one side of the equation, the same thing has to be done on the other side and being fair became the new reality.

Learning from mistakes:

This is an important tool. There has to be a scope for self - learning and discovery.
A student in 6th who enjoyed craft work, was provided with scissors and charts to work with different geometric shapes. Measurement of the two interior adjacent angles of a parallelogram were given as 60 and 120 degrees. Measuring it in the same direction produced a trapezium! Realizing his mistake, he corrected it immediately. We had struggled with the directionality aspect when I taught him how to use a protractor. He never repeated this mistake

The black board should have all levels of problems:
As a thumb rule, the black board is divided into three columns, the first column has basic level problems that is, problems which test the basic conceptual ideas of the topic, the second column consists of medium level problems (problems which consolidate learning) and 3rd column will have challenging problems (problems allowing exposure to advanced math, problems which introduce new ideas etcetera). Students are given the option of choosing problems from any column. If they complete all problems in their respective chosen columns, will move to the other columns. This ensures everyone in the class is engaged.
Problems are sometimes interchanged without the knowledge of the students. This will ensure that children who are unwilling to push themselves are actually working on higher level problems without knowing about it. Amazingly, this strategy has worked uniformly across grades.

Example:
Taking the topic of ‘Introducing Area and Perimeter ‘for the 6th graders, the black board would look like

<table>
<tr><th>Column !</th><th>Column 2</th><th>Column 3</th></tr>
<tr>
<td>a)What is the length and width of the shape?
c) What shape is this?
d) Can you find the perimeter?
e) And the area
f) Multiply length and width and see if you get the area? If you did, what’s the trick?

2) I am sure you can try finding area and perimeter of this one?
</td>
<td>1) What is the area and perimeter of the following figure?

2) Can you try this one?
</td>
<td>1) Find the areas of all the rectangle in the figure
2) Find the area of only the shaded region.
</td>
</tr>
</table>

The concept of Area and Perimeter, when introduced to the 6^{th} graders can be very confusing. The confusion or inadequate understanding of the concepts is also observed among the 10^{th} graders sometimes. They mostly go by the formulae.

Therefore, at the 6^{th} grade level, concrete objects like bricks tiles etcetera have to be used to cut, count etc. to make them develop proper conceptual understanding of Area and Perimeter. Thankfully, most of the text books now focus on building solid understanding. The questions above are chosen from one of the text books and the questions are modified to suit the requirement of the class.

This attempt is to build a stepwise understanding and gradually push the levels at which they are working.

Questions are staggered:
One of the main challenges of children with special needs is decoding a question. Frustration arises from the fact that they find it difficult to understand what to do. Therefore, it is essential to take them through the problem step by step.

The following example illustrates this concept:
There are **232** plants in a park. The Gardner plants **250 more** plants. **Estimate** the number of plants that are now in the park. This problem can be split into different levels like

(i) What do you understand by the word **'Estimate'?**
(a) Guess the answer
(b) Write the correct answer after calculation.

(ii) Which' **mathematical operation**' will you do?
(a) Addition (b) Subtraction (c) Multiply (d) divide.

(iii) What is your **estimated** answer?
(a)More than 400 (b) More than 450 (c) More than 400 but less than 450.

(iv) What is the **exact (correct**) answer you get?
(a)482 plants (b) 582 plants (c) 382 plants

Math in the Math lab:
Learning math in a math lab is different than learning math from the black board, so this year we have introduced the math lab. Students are found to be more engaged and exited to learn new board games. Some math related, some logic related, a few games are just fun to play with. Our observation so far has been that students who are least responsive in the class are showing a lot of interest in their math lab.
Hopefully, this will change their mindset to now think that Math is just like a game, and it is fun to play with numbers.

Conclusion : The above referred methods are perhaps basic, but they are certainly effective to form an inclusive environment and I think this is what we teachers strive for.

Do students in primary grades really understand the meaning of equal sign and can they be made to? Evidence from a learning module of an Intelligent tutoring system, Mindspark

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Should Indian students be expected to develop the understanding of the equal sign as 'relational operator' as early as grade 2? Are students as early as grade 2 (age 7+) ready for such abstraction? How can such students be made to understand the relational meaning of the equal sign? In this paper we shed light on a teaching approach to help establish the desired understanding of '=' sign as early as grade 2 and 3. Based on the pieces of evidence, we advocate a similar approach in Indian classrooms.

Keywords: equal sign; relational meaning of equal sign; teaching equal sign; Intelligent tutoring system

Students in primary grades encounter with an equal sign as much as they encounter with addition and subtraction sign. They would also see it in equations like 2 + 5 = 7, 13 - 4 = 9, 60 + 4 = 64, 64 = 6 tens + 4 ones, 5 × 2 = 2 + 2 + 2 + 2 + 2 etc. The common core standards in the US also expect students to understand the equal sign as a relational operator from grade 1. It has been repeatedly found by researchers that the understanding of equal sign, in significant cases, is wrongly constructed in students' mind (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Carpenter, Franke, & Levi, 2003; Rittle-Johnson & Alibali, 1999; Kieran, 1981; McNeil, & Alibali, 2005). The equal sign is perceived as a command to 'do calculation' or 'write the answer of' (operational understanding) rather than the mathematical symbol that conveys that what is written on the left and on the right to '=' are equivalent (relational understanding). The same misconception has been found among Indian students and analyzed in the previous study (Shah, Kanhere, Bhambhani, & Gupta, 2012). Studies have shown a 'strong positive relation between middle school students' equal sign understanding and their performance on solving linear equations', and they had showed that 'this relation holds even when controlling for the mathematics ability' (Knuth, Stephens, McNeil, & Alibali, 2006).

Extent of the misconception in Indian classrooms

The following item asked to grade 4 students of English medium private schools taking ASSET[1], shows that about 50% of the students (answering as C or D) are approaching the question with the 'operational meaning' of the equal sign in mind. They are either merely adding both the addends on the left side or adding all the 3 numbers given in the question.

220 + 380 = _____ + 400 What should come in the blank above to make the number sentence true? A. 200 B. 380 C. 600 D. 1000	Number of Students: 26267	
	Option A	**31.50%**
	Option B	9.90%
	Option C	33.70%
	Option D	14.30%

This wrong notion exists even among 16% of grade 8 English medium private school students in 5 metro cities of India (Shah, Kanhere, Bhambhani, & Gupta, 2012). Additionally, it was also found that Indian students struggle in comprehending the numerical equation which does not have the direct result on the right side. For example, 5 = 5 or 13 = 8 + 3 + 2 is more difficult than 3 + 4 = 7 or 3 - 1 = 2. More than 60% of 12022 grade 3 students do not find '5 = 2 + 3' as true as seen from the response data on an ASSET item. Around 50% of the 12022 grade 3 students consider '5

[1] A diagnostic test developed by Educational Initiatives taken by about 3.5 lakh students every year across different classes of English medium private schools in India and few Gulf countries with students of Indian origin

= 5’ as a wrong statement.

The reason for this distortion seems to lie in the fact that ‘a ± b = □’ is the very prominent question-type in the large number of Indian text-books, whereas the other question-types such as a ± b = c ± d, a = b ± d, c = c etc. are rare in these Indian text-books. The lack of exposure seems to be leading to an incorrect generalization of equal sign signalling the answer of the expression to its left rather than as balancing expressions on both its sides (relational meaning). A study (McNeil et al., 2006) finds that textbooks in the US rarely present equal signs in context most likely to elicit a relational interpretation—an interpretation critical to success in Algebra. Without the relational meaning of equal sign, students will struggle to make sense of ‘balancing both sides’ method of solving a linear equation (2x + 5 = 8 implies 2x + 5 - 5 = 8 - 5), numerical statements like 3 × 4 = 4 + 4 + 4, 2 + (8 + 7) = (2 + 8) + 7 (associativity) etc.

Our pedagogical approach to establish the relational meaning of ‘=’ sign

In order to reduce the extent of this misconception among students, two measures were identified. These measures are preventive in nature and focus on building the right conception in grade 2 and 3 itself, rather than remediating it in higher grades. The first one is establishing equations as number sentences which can be ‘True’ or ‘False’. Students, when presented with “non-action” sentences such as 3 = 3, found it wrong and ‘fixed’ that to ‘3 - 3 = 0’ (Behr, 1980). The other one is introduction to other relational symbols, such as > and <, with the equal sign. Dealing the equal sign with other relational symbols will make students internalize the notion of equivalence because ‘non-examples’ will also be present against them. The examples of both the measures are given below.

Measure-1	Measure-2
Identify the true statement(s). A) 3 = 3 B) 3 + 2 = 5 + 1 C) 4 = 7 - 3 D) 5 + 5 = 10 Fill the blank to make the statement true. 3 + 2 = ___ + 1 A) 2 B) 4 C) 5 D) 6	Write the correct symbols (=, >, <) in the blanks below. A) 5 [___] 3 B) 3 + 2 [___] 5 + 1 C) 10 - 6 [___] 7 - 3 D) 4 + 4 [___] 8 + 8

A learning module in an intelligent tutoring system (ITS), Mindspark[2], with 54 questions was designed by two of the authors to establish the appropriate conception of the equal sign (in triplets of 18 unique question formats arranged in sequence)[3]. In this learning module, only the first measure was used. The questions were intentionally designed, not to assess, but to make students think in a particular way and lead them to understand the relational meaning of ‘=’ sign. No direct teaching instruction, such as video or text, were included. The carefully designed questions and their sequence, instant feedback with worked out solutions showcasing a strategy, were the means to enable learning.

It is seen that in general students lacking the desired understanding of equal sign also lack the understanding of numerical expressions—not understanding that same number or term can be expressed by many different expressions. ‘Even in the absence of the “=” symbol and the box or the blank, the numerical expression 2 + 4 (not the numerical statement 2 + 4 = □) serves as a stimulus to do something’ (Behr, Erlwanger, & Nichols, 1980) and not perceived as one out of the many expressions for the number 6. For example, terms like 100 - 99, 2 - 1, 99/(90 + 9) and 3^0 are expressing the very same number; they do think that on solving each of this, they will get 1 but do not think these expressions represent 1. Therefore, the learning module in Mindspark first establishes this by the questions similar to what is shown in figure 1. Once it is established that 4 + 2 and 3 + 3 are equivalent, we introduce the symbol ‘=’ that denotes equivalence as shown in figure 2.

[2] Mindspark is an online personalised and adaptive learning platform where students with paid subscriptions by Indian English medium private schools learn along with usual classroom teaching.

[3] For each unique question, a student is given 3 attempts. If the first question is answered wrong, explanation, why the Q was marked wrong, is provided and then the student is presented with a similar (copy) question.

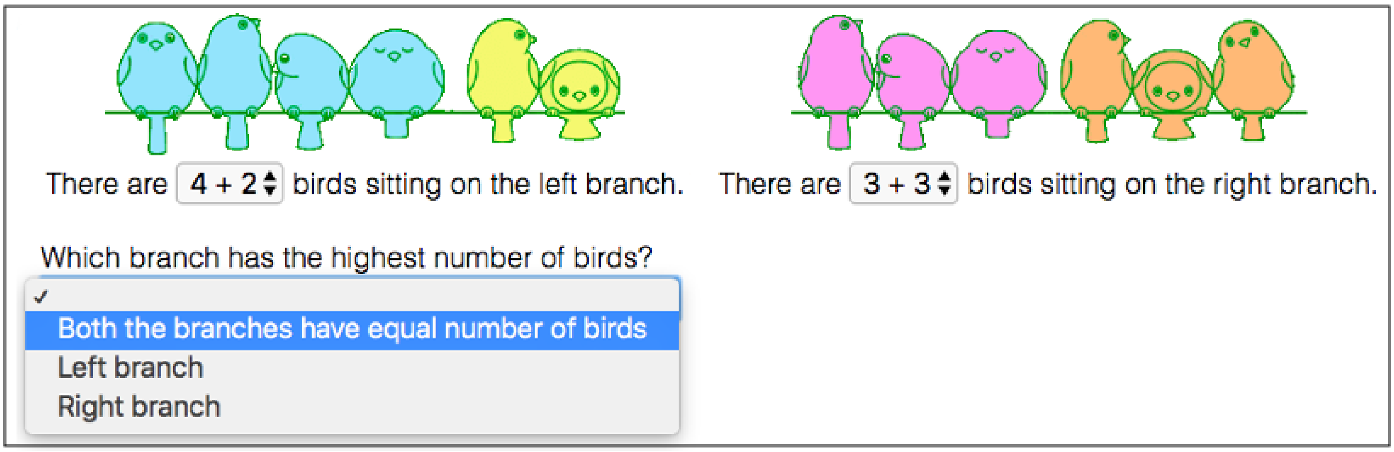

Figure 1: First a student selects 4 + 2 and 3 + 3 from the drop-down options, then she is asked to compare the both to make her realise that the same number is represented with different expressions. Visuals of birds are used with the hope that they will answer the question without actually computing 4 + 2 and 3 + 3.

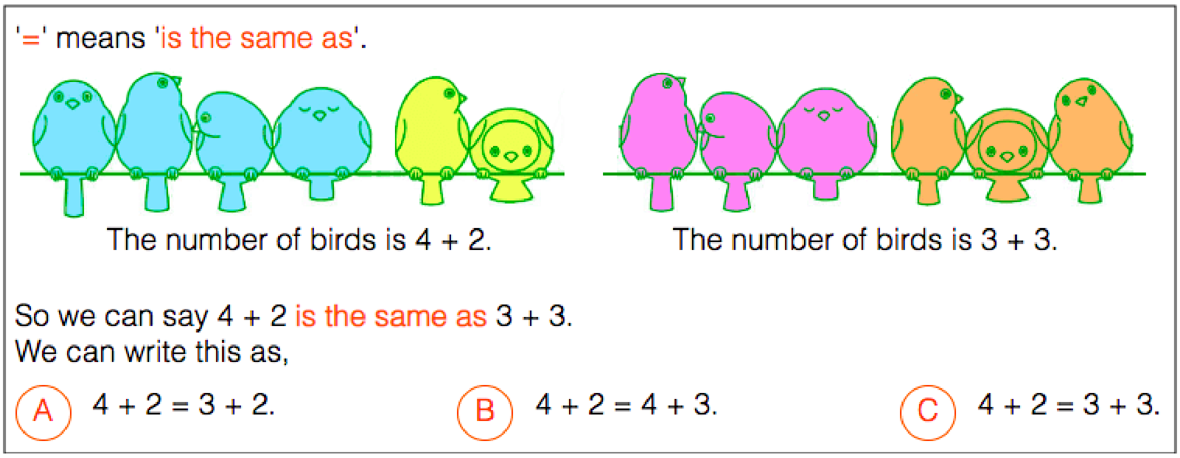

Figure 2: The symbol '=' was introduced.

Later, when students got comfortable representing situations by a number sentence having '=' sign, the module introduced the questions asking for the truth-value of the similar numerical statements. In the explanation of these questions, ten-frames are used as they clearly show the numbers being represented and, by subitizing, one can compare the quantities visually. This way, the questions in the module encouraged students to realise say 4 + 5 = 3 + 3 is a false statement because 4 + 5 and 3 + 3 are representing different numbers (number of dots), rather than only thinking that they are not equal because upon finding both the sums they will get different numbers.

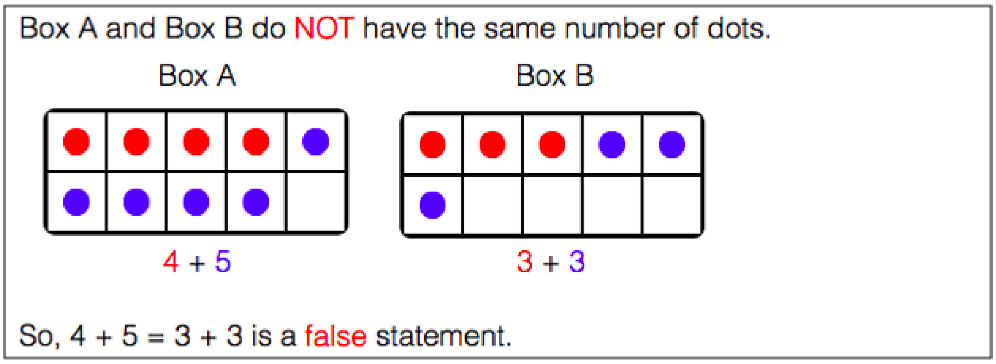

Figure 3 : An example from the module using ten-frames to explain why the statement is false

In the module, the questions asking for the truth-values were placed before the 'fill in the blank' questions in order to make students think 'how to make a number sentence true' while filling the blanks. The module again used the same context of birds and visual aids to introduce such question type as shown in the figure 4 below. Besides number pattern, the questions shown in figure 5 build foundation for algebraic reasoning. After giving few practice questions on this, the similar flow of questions were used for statements having the subtraction sign.

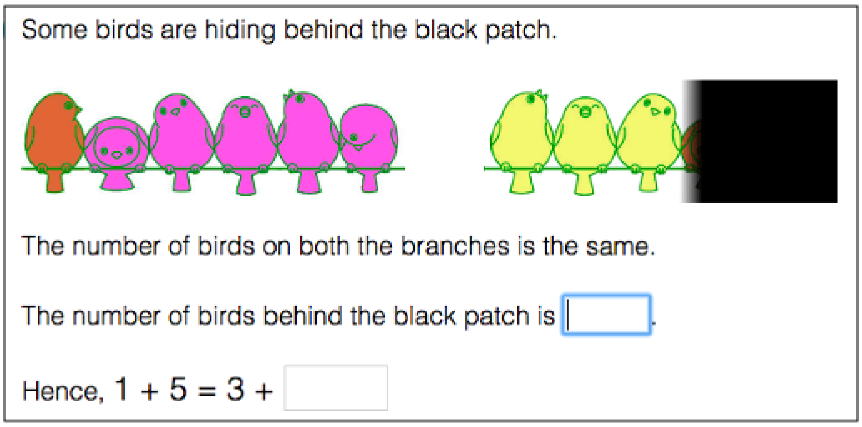

Figure 4

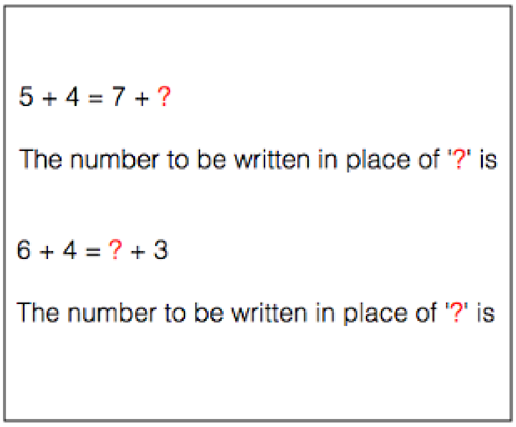

Figure 5

Results

We studied the student response data of about 650 students of grade 2 and 3 of English medium private schools who completed the learning module targeted to teach 'relational meaning of equal sign' through the ITS, Mindspark. The accuracies on the questions below show that students of grade 2 and 3, if provided with a well-designed content, can build the desired understanding of the equal sign. Based on this idea and the fact that a high number of Indian students do not understand the relational meaning of the equal sign, we advocate the teaching of the relational meaning of equal sign from grade 2 onwards with increasing complexity.

A sample question in the learning module	% of student who answered 2 or 3 out of the 3 similar questions correctly
4 + 6 = 5 + ?	76.5% in Grade 2 (N = 289) 78.4% in Grade 3 (N = 356)
5 + 7 = ? + 6	74.3% in Grade 2 (N = 152*) 76.0% in Grade 3 (N = 96*)
Which of the following statements is FALSE? A 6 - 2 = 2 + 3 B 5 = 2 + 3 C 5 = 7 - 2 D 8 - 2 = 3 + 3	62.3% in Grade 2 (N = 289) 66.6% in Grade 3 (N = 356)

*lower N as the question was added in the module later

Discussion and Conclusion

Alternatively, one may also use the actual balance, shown in figure 6, to establish the sense of equal sign as 'balancing numerical expressions on its either sides'. Some students, while solving questions like $3 \times 4 + 2$, use the equal sign as a step-marker and write incorrectly as $3 \times 4 = 12 + 2 = 14$. The balance-model, if established before such questions were given, will deter students from writing like this and be mathematically correct in writing. Otherwise one will not only continue to have the misconception but also run the risk to reinforce it further. Also questions like $95 + 50 = 94 + \square$ should be exposed to the students and they should be encouraged to solve by balance-model thinking (94 is 1 less than 95 so the number should be one more than 50 to balance both sides) in order to enhance flexibility with numbers.

How many violet blocks should there be to keep the balance?

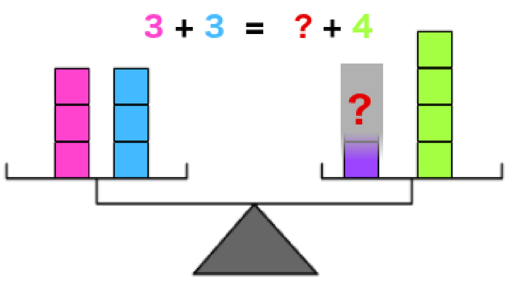

Figure 6

Chinese textbooks have adopted this approach and they typically introduce the equal sign in a context of relationship and interpret the sign as "balance", "sameness", or "equivalence" in grade 1 after introducing numbers up to 10. Because of this approach, Chinese students show near zero presence of misconception related to the equal sign.

Chinese textbooks have also ‘introduced “=,” “>,” and “<” together with pictorial and written representations’ (Li, Ding, Capraro, & Capraro, 2008) as suggested in measure 2.

Results from the data of the learning module in Mindspark shows that, if the design of the content is research-based and robust, students of primary grades can grasp the desired meaning of equal sign. We advocate the teaching of equal sign from grade 2 onwards with the approach illustrated.

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DIFFICULT TOPICS IN MATHEMATICS CURRICULUM

Mathematics is an indispensable part of school curriculum, not only this it is an indispensable part of our daily life. It is not only important to learn it as a subject but it also serves as a tool to learn and understand other subjects. Most of the stake holders belief that it is one of the most difficult subjects, but as a teacher of Mathematics I belief it is just our notion about this beautiful subject. We teachers and parents first need to change our mind set about the subject and then only we can make our off springs to enjoy it. Through this paper I would like to through some light on topics which students find difficult in their curriculum and why they find these topics as difficult.

Introduction:

Mathematics concepts are vast, interrelated and possess interconnected elements. The interrelationship of Mathematical concepts can be identified in the use of elementary operations of division, ratios, percentage, addition, subtraction, translation of word problems and use of symbols across mathematics. Students experience difficulty in constructing meanings of symbols.

A student not able to get good marks or grades in a collection of mathematics problems is not completely showing the inability of a student, but also difficulty in understanding certain topics. The reason for the same is student's persistent hitch makes procedural approach to cognition of a mathematics concept a hideous task. Robertson and Wright (2014) stated that students generally have intrinsic difficulty in mathematical reasoning, mathematical ideas and understanding basic mathematical concepts.

The conceptual knowledge in mathematics requires adherence to an algorithm that leads the solver through a correct process to a correct answer. During instruction, students should be allowed to actively participate in each step of a problem solving algorithm for formalization and effective practice. Some students' difficulties can be attributed to inappropriate representation and handling of problems. Mathematics phobia among students and their attitude towards mathematics learning have been identified as major hindrances in learning mathematics. Attitude is concerned with an individual's way of thinking; acting behaving has serious implications on the learner. The student attitude towards a learning process whether innate or emulated, reshapes his behavior in the classroom and an emotional disposition towards mathematics.

Learning is influenced by many factors, which can be cognitive or affective. Here, we are concentrating on affective side. Hart (1989) defined attitude towards mathematics as a complex of negative or positive emotions that associated with mathematics, individual beliefs towards mathematics and their behaviour associated with mathematics.

Beliefs are one of the main areas of affective research since 1990s. Beliefs can be defined as implicitly or explicitly held subjective conceptions students hold to be true, that influence their learning.

Mathematics is a subject that causes many negative emotions. One of the main challenges to mathematics teacher is to make positive attitude in students toward learning mathematics. Teachers with positive attitude can stimulate favourable attitude in their students. Therefore, teachers should be aware of students' effective beliefs and inter relations of those in learning mathematics so as to employ more effective strategies in teaching and to improve students' mathematics learning by reducing their negative beliefs.

Generally it is found that students find Algebra more difficult as compared to Geometry, whereas some students find Arithmetic as the easiest. Students find Algebra difficult perhaps of its abstract nature, it involves variables etc. On the other hand Geometry is more application based, its usage can be found in our daily life. Geometry is having theorems whose application students can find in solving the questions. On the other hand if we talk about Trigonometry and Statistics, students able to find their usage in daily life, both these topics are practical in nature and students able to learn by connecting it to their routine life as well.

To elaborate on the above, here I would like to share my class room experiences, about the difficulties generally students face in solving problems related to Algebra as compared to other topics. Students are generally not able to comprehend the statements or wrongly interpret the statement, due to which they are not able to perform good. Students whose language skills are poor they are getting confused with words like double or twice, where they suppose to multiply they add or divide. Some time they use the same variable for two or more different parts of the same question or they use the same symbol which is already given in the statement, which ultimately leads to poor performance in the subject. So students start disliking the subject.

Students dislike mathematics as they perceive it as a difficult subject. Also, the association of these two with "I can/can't "do math (self-efficacy) is significant. Perception of math as a difficult subject is associated strongly to lower self-efficacy than disliking of the subject. This finding supports very much the findings of Zan and Martino (2008) that students like mathematics as they can do it and dislike it as they can't do it. Liking of mathematics is associated with more positive affects like interest, Self-efficacy is a person's perception about his ability to reach the goal (Bandura, 1977). Self-efficacy does not represent one's ability, but his beliefs; it affects achievement through the selection of task and effort. Expectancies for success is defined as one's beliefs about the success of his or her performance on an upcoming task.

Another reason why students consider Mathematics is a difficult subject is, due to aversive teaching style, difficulty in following the instructions, difficulty in understanding the subject and difficulty in remembering its equations and ways to solve problems. Mathematics has some inherent difficulties due to its abstract and cumulative nature. So students requires a firm foundation, they may not be able to learn new things without previous knowledge. For many students expectancy about the difficulty of maths high and personal value attached with math is low. In the case of these students, the chance for developing an avoiding or escaping tendency will be high. Students are accepting the utility value of mathematics, but they haven't any personal value attached with mathematics. So, though they do not like mathematics they may choose to study because of its practical value. But when a task is difficult to them chance for avoiding that task will be higher. One of the positive beliefs students hold that they can do better if they try hard; they are accepting the value of effort.

Students' report and teachers' perception indicate that major cause for mathematics being difficult for students is lack of previous knowledge. Without relevant previous knowledge it is difficult and even impossible to learn mathematics in higher classes. Teaching students without prior knowledge promotes mechanical or rote learning. Gradually students tend to believe that they are not fit to learn mathematics or they would not able to learn mathematics. Here students attributing failure or backwardness in mathematics to an internal, stable and uncontrollable cause, but actually it were an internal controllable reason. So teachers should spend more time for making relevant prerequisites and to make them aware that problem lies with their learning strategy. Otherwise difficulty becomes progressive and as they move on to higher class students' achievement in mathematics will go down.

Thus reciprocal relationships exist between every attitudinal measure and mathematics achievement, and the feeling of enjoyment directly affects mathematics achievement. Teachers need to use effective ways to motivate students to learn mathematics regardless of student difficulties. Teachers can contribute to improve students' liking of the subject by improving students' affective beliefs.

गणित कक्षा में बच्चों से संवाद के मौक़े: डाटा हैंडलिंग (Scope of dialogues in Math classroom: Data handling)

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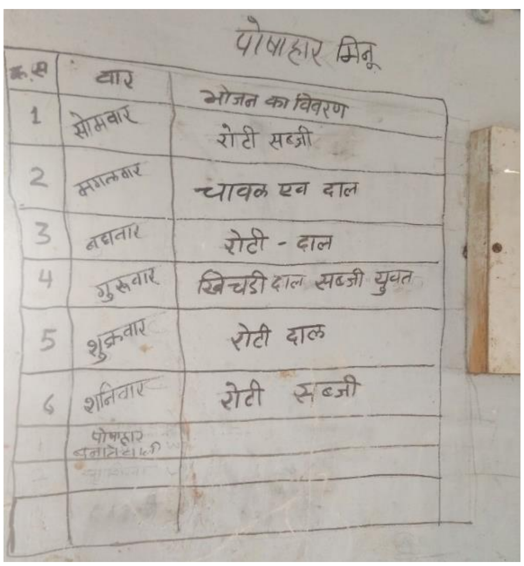
गणित की कक्षा में (ख़ास कर प्राथमिक कक्षाओं में) बच्चों से संवाद स्थापित करना एक बड़ी चुनौती मानी जाती है और है भी । कुछ शिक्षकों को छोड़ दें तो गणित की कक्षा में अमूमन देखा जाता है कि शिक्षक बच्चों से एकतरफ़ा संवाद करते हैं जिसमे बच्चे श्रोता मात्र बन कर रह जाते हैं और बोर्ड पर हल किये हुए सवालों को नोट करते नज़र आते हैं। वैसे तो पूरे गणित को ही कक्षा में रुचिकर बनाया जा सकता है और बच्चों को बोलने व अभिव्यक्ती के मौके दिए जा सकते हैं। लेकिन यहाँ पर केवल "डाटा हैंडलिंग" यानी कि "आंकड़ों का प्रबंधन" पर ही विचार किया जाएगा कि यह कक्षा-कक्ष में बच्चों को किस प्रकार अभिव्यक्ति के अवसर प्रदान करता है । तथा प्रत्येक बच्चे को प्रतिभाग करने के मौक़े भी देता है और साथ ही उनके अंदर कुछ संवैधानिक मूल्यों का भी विकास करता है ।

प्राथमिक स्तर पर "आंकड़ों का प्रबंधन" गणित पाठ्यपुस्तक का एक अहम हिस्सा है जिसका उद्देश्य बच्चों में आँकड़ों को व्यस्थित करना, उसका विश्लेषण करना तथा उससे कुछ निष्कर्ष/जानकारियाँ निकालना है। एक स्कूल में चौथी कक्षा के बच्चों के साथ डाटा हैंडलिंग पर कार्य कर रहा था, उसी कक्ष की दीवार पर मध्याह्न भोजन का "पोषाहार मीनू" बना हुआ था जिसको आधार बना कर मैंने बच्चों से बातचीत शुरू की ।



बच्चों को पोषाहार मीनू की तरफ़ इशारा करते हुए इसके बारे में पूछा तो उन्होंने ज्यों का त्यों पढ़ दिया, फिर मैंने दिए गए मीनू से निम्न सवाल किये-

a) बुधवार को खाने में क्या मिलता है? b) रोटी सप्ताह में कितने दिन मिलती है? और वह दिन कौन-कौन से हैं? c) दाल कितने दिन मिलती है? d) सप्ताह के प्रथम दिन व आखरी दिन में क्या मिलता है?

बच्चे कुछ बताने में संकोच कर रहे थे फिर मैंने संगीता नाम की बच्ची को खड़ा किया। संगीता थोड़ा समय लेते हुए सारणी को देखकर बुधवार के भोजन विवरण के बारे में बता देती है । अगले सवाल को भी कुछ बच्चों द्वारा कर लिया गया, बच्चे अब थोड़े सहज हो चुके थे । उस कक्षा के गणित-अध्यापक भी कक्षा प्रक्रिया में भाग ले रहे थे, चूँकि

भोजन के विवरण में कुछ त्रुटी थी जिसपर शिक्षक ने बच्चों से कहा की सर (यानी मुझे) को बताओ कि फल किस दिन मिलता है । बच्चों ने फिर बताया कि उन्हें प्रत्येक सोमवार को फल भी दिया जाता है, जिसके बाद मैंने बोर्ड पर एक अलग सारणी बनाई और बच्चों से ही इसे सुधार करने को कहा ।

सूचनाएँ एकत्रित करना और उन्हें व्यवस्थित तरीक़े से रिकॉर्ड करना (ताकि वह किसी दूसरे के लिए स्पष्ट हो) ही डाटा हैंडलिंग यानी की आंकड़ों का प्रबंधन है ।

आगे बच्चों से एक **गतिविधि** कराई जिसमे उनके द्वारा पसंद किये जाने वाले कुछ फलों के नाम बताने थे। बच्चों ने अनार, केला, अंगूर, सेब, आम, पपीता के नाम बताए । एक चार्ट पेपर पर 6 कॉलम बना कर इन 6 फलों के नाम लिख लिए गए और उसे दीवार पर बोर्ड के बगल में चिपका दिया गया । बच्चों को एक-एक चिट दी गई, चिट पर बच्चों को अपना नाम लिखना था और उसे अपने पसंदीदा फल वाले कॉलम में चिपकाने थे, बच्चे रूचि के साथ अपने-अपने चिट में गोंद लगा कर उसे चिपका रहे थे । यह एक ऐसी गतिविधि थी जिसमे प्रत्येक बच्चे शामिल हो रहे थे ।

जब सभी बच्चों ने इस क्रिया को कर लिया तब चार्ट पेपर पर एकत्रित किये गये सूचनाओं के आधार पर कुछ प्रश्न तैयार किये गए और बच्चों से पूछा गया जो निम्न हैं ।

1. सबसे अधिक कौन सा फल पसंद किया जाता है ?
2. सबसे कम पसंद किया जाने वाला फल कौन सा है ?
3. कितने बच्चों द्वारा आम और अंगूर पसंद किया जाता है?
4. किस फल को केवल एक बच्चे ने पसंद किया है ?
5. रमेश नाम के बच्चे को कौन सा फल पसंद है ?

उपरोक्त सवालों में कुछ बच्चों ने गलती भी की जिसे बाकी बच्चों द्वारा ही ठीक किया गया, बच्चों से और भी नाम लेकर पूछा गया कि किसको क्या पसंद है । मेरी मंशा थी कि यह आंकडें बच्चों में चर्चा का माध्यम बने जिसके लिए इसको कक्षा की दीवार पर चिपकाना बेहतर समझा । चार्ट को दीवार पर चिपकाने के बाद मैंने बच्चों का अवलोकन किया जिसमे पाया कि उनके लिए यह पता लगाना आसान था कि किस बच्चे को कौन सा फल पसंद है, किन बच्चों को एक जैसा फल पसंद है आदि । मैंने बच्चों के साथ आकड़ों के विश्लेषण की कौशलता पर और काम करना चाहा, मैंने बच्चों से चार्ट देखकर बताने को कहा कि कौन सा फल सबसे ज़्यादा पसंद किया जाता है । जिसके जवाब में बच्चों ने आम का नाम लिया, यहाँ पर मेरे लिए एक अच्छा अवसर था जहाँ आम की विशेषताओं पर उनसे बात करता लेकिन समय के अभाव में आगे बढ़ा । मैंने चर्चा को और आगे बढ़ाया और उनको ध्यान दिलाया कि आम फलों का राजा भी होता है, राजा शायद इसलिए होता है कि इसे ज़्यादातर लोग पसंद करते हैं, आदि । जैसा मैंने शुरू में वर्णन किया कि गणित की कक्षा में बच्चों से संवाद स्थापित करना एक बड़ी चुनौती मानी जाती है और है भी, इस चुनौती पर काबू पाने के लिए कुछ गतिविधिया और खुले प्रश्न को तरजीह देना कारगर हो सकता है जिसमे डाटा हैंडलिंग की अपनी एक अलग भूमिका है । आज सूचनाओं का युग है जिसमे हमारे पास अनेक आँकड़े/डाटा हैं, जिसका

प्रबंधन, विश्लेषण व उस आधार पर कोई निर्णय लेना हमारी आम दिनचर्या में शामिल है जैसे ऑनलाइन खरीदारी करना, चुनाव में अच्छे उम्मीदवार को चुनना, आगे संभावनाएँ तलाशना आदि । इन सारी प्रक्रियाओं में डाटा हैंडलिंग की कौशलता हमें बहुत ही मदद करती है । इसके अलावा यह शिक्षा के बड़े उद्देश्यों की भी पूर्ति करता है जो संवैधानिक मूल्यों से संबद्ध हैं यानी तर्कशील व स्वतंत्र विचारक नागरिक तैयार करना । तर्कसंगत बातें करना तभी संभव होता है जब किसी के पास उस विषय पर पूर्व जानकारियाँ हों या यूं कहें की आंकडें/डाटा उपलब्ध हो और वह उन आंकड़ों का इस्तेमाल करते हुए अपनी बात रख्खे । जब बच्चों के अन्दर ऐसे कौशलों का विकास होता है तो वह किसी भी बात को यूँही नहीं मान लेता है बल्कि अपनी जानकारी/आँकड़ें के बिना पर उसकी सत्यता और वैधता की जांच करता है और उससे सहमत होता है या ख़ारिज कर देता है । आंकड़ों का प्रबंधन ऐसे कौशलों के विकास में एक महत्वपूर्ण भूमिका निभाता है ।

ऐसे कौशलों के विकास के लिए प्राथमिक कक्षाओं में "आंकड़ों के प्रबंधन" संबंधित गतिविधियों की अपार संभावनाएँ हैं । कक्षा की दीवार पर बच्चों के नाम और उनके जन्मदिन से जुड़े आकड़ें, कक्षा में बच्चों की हाइट के आँकड़े, आदि प्रदर्शित करना बच्चों के लिए बहुत कारगर हो सकता है, हम बच्चों को स्वयं इस कार्य में आगे रखकर करवा सकते हैं । बच्चे इससे कक्षा में अपनापन महसूस तो करेंगे ही और साथ ही अपने मित्रों के जन्दीन पर उन्हें बधाई देना, एक साथ मिलकर उसका उत्सव मनाना और साथ ही नए साथियों के आने पर चार्ट में उनका नाम और जन्मदिन जोड़ना जैसे कार्यों को करवाया जा सकता है जिसमे निसंदेह शिक्षक की अहम् भूमिका होगी । इस कार्य से बच्चों की रूचि बनाए रखते हुए बच्चों के अन्दर अनेक परिप्रेक्ष्य और कौशलों पर काम किया जा सकता है ।

EXPERIENCES OF TEACHING VECTORS IN INDIAN PRE-UNIVERSITY CLASSROOMS: AN ACCOUNT BY TEACHERS

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ABSTRACT

The topic of vectors, usually introduced at the pre-university level, is a powerful mathematical modelling system with wide applications in physics, and engineering, given its amenability to formally describe the direction. However, it has been considered a very tough and complex topic to teach and learn by the teachers and students. There has been a lot of literature in both physics education (Aguirre, 1988; Barniol & Zavala, 2010, 2014) and math education (mostly under the umbrella of linear algebra) (Dorier, Robert, Robinet, & Rogalski, 1999; Dorier & Sierpinska, 2002; Hillel, 2000) acknowledging the difficulties that the students face with this topic. In this connection, here, as math and physics teachers teaching this topic to the students of grade 11 (pre-university level) in Indian classrooms, we would make an attempt to highlight certain experiences we encounter in our teaching of vectors and present pointers which could be of benefit to the teaching-learning practices on the ground.

Treatment by the curricula

Vectors in many mainstream Indian curricula are presented in both physics and math curricula though are introduced first in the former. We broadly cover the topics of definition, properties, addition, resolution and products of vectors. The mathematics curriculum includes proving geometric theorems and problem-solving (like 3D coordinate-geometry). The physics curriculum uses them mainly in the applications of mechanics, electricity and magnetism etc.

In 11th grade Maharashtra state board textbooks, the definition provided for a vector as a quantity with direction and magnitude is incomplete. In our experience, we faced questions on the quantities like current and time, which do have a sense of direction. We handle this by completing the definition by adding the need to follow the triangle of addition. Further, the ordering of topics itself is often problematic in the classrooms. A more detailed textbook analysis can be found in the work of Karnam and Sule (2018). Besides vectors, topics like the potential in physics employ integral calculus even before it is formally dealt with; this could be avoided using summation.

There is a gap in mathematics and physics treatments of vectors and the issues it could raise are acknowledged by Dray and Manogue (1999). Citing examples from our experience, the notion of free vectors gets tricky, especially given the treatment of vectors in mathematics as position vectors. When dealing with the parallel translation of vectors, students wonder how they remain the same, even when we shift them to other quadrants, confusing it with position vector. We explain this by noting that if the components are same, then the vector remains the same. Further, the ideas of collinear vectors and coplanar vectors could be more coherently presented. Resolution and components also have different treatments in physics and math. And even the representations (eg: **a, A, AB**, read with arrow heads on the top) could be standardised at least in the beginning stages or explicitly the difference needs to be acknowledged. Given the differences in the objectives and the course, vectors need to take in physics and mathematics, these differences are perhaps inevitable, but they definitely cause difficulties to the students. So, a careful thought needs to be given to this at least at the introductory levels.

Some tricky situations due to the nature of the topic

There are many situations, and given the limited space, we highlight a few. Explaining unit vector is something that we struggle to appreciate. We define it as a vector with a unit magnitude in the direction of a given vector. For example, they understand unit vector along a displacement vector of 5 meters due east (split as magnitude and direction), when explained by removing the magnitude and retaining a direction of one unit due east. However, students find it difficult to completely appreciate the mathematical significance of unit vectors while writing rectangular components, given the above definition. We address these partially by taking examples like how a unit vector along (3i+4j) is obtained by dividing both the components by the magnitude, 5, and get $\frac{3}{5}$ i+ $\frac{4}{5}$ j, whose magnitude can be found to be 1.

Explaining cases where the vector nature of quantities is inconsistently presented in the textbooks when dealing with infinitesimals or instantaneous quantities gets trickier, as there is an understanding of vectors as well as calculus needed. For example, a finite angular displacement is introduced as a scalar whereas instantaneous angular displacement is considered a vector. Further heuristics of problems solving in physics gets tricky. For example, in a free body diagram of a simple pendulum, we resolve the weight and of a conical pendulum, the tension, appreciating which is often difficult for the students.

Visualising is another serious limitation with vectors. It is very highly pronounced in the case of vector product, where they need to imagine in 3D. In general, they are very comfortable to perform operations (including vector products) when given in *ijk* form mechanically. We constantly struggle with in making them imagine. We often use resources within the classroom like the corners of the room, the table or use familiar objects like pen or ID card strings among others to explain and assist them in imagination. But there is a limitation to what we can do. This problem gets more complicated when in topics like the superposition of waves we need to draw and show waveforms with different frequencies on the board, and we are highly handicapped.

Conclusion

Given the general state of education, especially in pre-university level in India, when the students are subjected to high levels of competition to crack entrance examinations, there is a little motivation for them to appreciate, understand and learn beyond what is needed to meet the needs. These conditions among the other factors from the curriculum and the nature of the topic, as we presented in previous sections, worsens the situation for a student. A possible way ahead could be equipping teachers with the skills needed to address the trickier situations in the classrooms and using technological means to address the visualisation problems. A major revamp of the textbooks keeping the student at the centre could be another productive pursuit.

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Learning Mathematics concepts through Projects and EBD(Education by Design) methodology

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ABSTRACT

Context

We are a team of engineers who run STEM (Science Technology Engineering Mathematics) land – that are rural STEM centres in two outreach schools of Auroville – Udavi School and Isai Ambalam School. Both schools aspire towards holistic development of the child and the managements are progressive. The children attending come from villages surrounding Auroville. Udavi School follows the state board syllabus and we work with ~56 children from 7th to 10th intensively for 6 hrs/week for all their Mathematics (Math) classes. Isai Ambalam School follows the central board syllabus and we work with ~71 children from 3rd to 8th grades intensively for 6 hrs/week during the Environmental Sciences (EVS) and Math classes. In demographics, the occupation of parents in both schools is in unorganized labor e.g. masons, painters, agricultural labor and schemes providing rural employment. The predominant community accessing Udavi School is MBC (Most Backward Caste) and that accessing Isai Ambalam School is SC (Scheduled Caste).

At STEM land children learn Mathematics, Electronics, 3D Printing, Programming (in Scratch, Alice, Geogebra), Mindstorms (Robotics) and play strategic games that enhance logical thinking. The children take responsibility of their learning and plan their goals each week related to their curriculum and beyond it. This self-directed learning is based on Sri Aurobindo's first True principle of education (Aurobindo, 1910); "Nothing can be taught". They create projects that represent their mastery over concepts they learn and can share following constructionism (Papert, 1986). They and work individually, in pairs or peer groups and ask for support from facilitators when they need it. With younger children we work on real life projects that impact their surroundings.

In this paper, we are sharing our experience as practitioners, of how different children learned Mathematical concepts such as sets, algebra, measurements, ratios and percentanges at STEM land. We also look at how technology (both physical e.g. building/construction and virtual e.g. programming) was effective in aiding this process. We will take some case studies.

Case studies

Sets and Algebra: A few children from 10th grade in Udavi School had built a physical game with a chart representing Venn diagrams (where each Venn Circle represented a rule e.g. one for color and one for shape). There were tokens (e.g. red circle) representing characteristics that the player can place and check if it fits in a section of the Venn diagram. The goal is for the player to determine the original rules of the Venn Circles.

This game inspired Diva, a 9th grader the built this game in Scratch. Scratch is a visual programming

language that allows children to build interactive games, animations that are used extensively at STEM land by children to create projects. It requires children to break down a problem into simple enough components so they can be understood by the computer through instructions. We have seen that this helps children learn concepts in a more concrete fashion (Ranganathan s, et al 2015) while improving problem solving, logical reasoning, etc.

In this case, Diva realized that in order for the computer to understand which region of the Venn Diagram was being accessed he needed to divide the Venn diagram (for 2 circles) into A-B (in A and not in B), B-A, A∩B (in A and B), and U-AUB (outside the two circles). Even though visually it looked like a single picture it was in fact composed of 4 different pictures (or sprites). This also helped him understand the different regions of the Venn diagram better. Creating these separate shaped areas allowed him to use sensing of a new object (token) to determine if it belonged to this area or not. He made the game first with a single rule which was not interesting for him to play and he then generalized it so the computer would randomly pick the rules and it would be a challenge for him too. Creating games like these not only help children understand the concepts better, but in testing it add to the rigor of learning the concepts well. Just as Diva had been inspired by a game made by others, 8th graders were inspired by Diva's game to understand how it works and learn about sets even though they did not yet have it in their syllabus. At STEM land we have sessions where children share their projects once a week to encourage such learning.

To be capable of doing the kind of project above children at STEM land like Diva learn scratch by making smaller projects, through peer interactions and through interactions with facilitators. This is generally need based and strong in some areas and weak in others. To make their skills in all areas rigorous we conduct organized courses at STEM land. These courses were based on developing basic understanding of various capabilities of scratch programming to allow children to create more complex code.

After completing one such course a program relating algebraic identities with area was built by a few 8th graders. For example, Jan made a program that drew (a+b+c)2 as three squares i.e. a2,b2,c2, and 2ab, 2bc, 2ca as areas of rectangles. Images such as these were also available in the text book. The images in the book, however, were static and used a fixed value of a, b, c. Then as they changed the values of a, b, c to be variable and random they could see the different sizes still held the fundamental shape of a rectangle to get a sense that these were really variables.

Pond EBD: The second case study is in Isai Ambalam School with real life projects. The children had created a pond (Iyyanarappan a, et al 2019) through which they had learnt measuring length, perimeter and calculating circumference, volume of the pond. While creating mortar they learnt proportions and ratios in order to create the right mixture of sand and cement (3:1). However the pond developed some cracks due to roots from trees nearby. Children came together and built a frame in the shape of the pond and through this they learnt to bend metal rods(6mm and 12mm) at specific angles such as 900 450 etc. They also learnt unit conversion from inches to cm for buying the appropriate rods and to cut them in right dimensions. Once the frame was done, they mixed RCC (ratio 1:2:3; cement: granite gypsum: sand) and poured into the structure filling all the rods and finally smoothened it. Through this process they learnt angles and frames as well as ratios and proportions with more than two quantities. We observed children who are less engaged in academic classes are enthusiastic in building with their hands. In this example we have looked at building technology as a way for children to learn.

Shop EBD: In Isai Ambalam School the 7th and 8th graders had difficulty understanding profit and loss. On looking at the prices of stationary in the shops around they found that the price varied and the local shops in the village were charging too much. They also noted that it was not always easy for the young children to have access to shops for small items they needed like pencils, erasers, scales, notebooks that their parents were not always able to provide at the required time. To move from dependence to interdependence at the school they started a small shop of needful things during recess. They raised the investment for this shop as a cooperative among children and teachers and bought items in bulk from Pondicherry and found that they could provide the same items to the children at a cheaper price than that of any local shops. They started the shop in July and it has been successfully running accounting for all costs including transportation. This stationery shop not only taught the children about profit and loss, but also keeping stock, writing receipts and understanding how shops price materials.

CONCLUSION

Creating projects provides children with a way to demonstrate their learning and offers alternatives to examinations as a form of assessment. Furthe r, it offers an opportunity for self-evaluation and constant progress. Through real life challenges aided by physical technology e.g construction, children can learn a lot even beyond their curriculum and connect theory to practice.

Visual technology like computer can be used for programming and helps children learn conceptual ideas better as they need to break it down into small bites of step-by-step instructions for a computer. Creating visual representation also helps them understand abstract concepts. Learning programming builds rigor as they also repeatedly test their programs.

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Relating teacher professional development and development of students' mathematical understanding

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Introduction

Teachers' have been engaged in inservice teacher professional development program with the aim of creating a reflective approach towards classroom teaching and help teachers in trying out new and innovative ideas for teaching. Often, the professional development opportunities are provided through workshops and follow up in the classroom is not done. In contrast, Connected Learning Initiative (CLIx) designed and implemented a continuous professional development opportunity for in-service teachers. In this paper, we discuss a classroom experience of a teacher (second author- Amarjyoti Sinha) using the case of a lesson on fractions in which she used the ideas which she had become familiar with while engaging in the continuous professional development program. The studies on professional development efforts rarely report from the perspective of the teacher who are engaged in these opportunities. The description of this lesson and the commentary by the teacher on what prompted her to select the tasks and draw connections between them in her teaching reflects how she sees her practice as a site for exploring the ideas in the classroom with the students while making effort to connect the mathematics with daily life of students. The selection of tasks and her response to students helps in making the connection between the ideas discussed in the professional development context with the context of classroom teaching and eventually to the level of understanding that students are developing as a result of use of these tasks.

Background about the teachers' participation in the CLIx professional development initiative

CLIx is a large scale ICT in education intervention which is trying to demonstrate scalability and sustainability by taking a systems approach for adoption. vision of using ICT as a platform to groom students as producers of knowledge that the Tata Institute of Social Sciences (TISS) in collaboration with MIT, Boston, Eklavya, HBCSE (Homi Bhabha Centre for Science Education), a number of other strategic national partners and state governments of Rajasthan, Chattisgarh, Mizoram and Telangana, developed the design for CLIx. CLIx changes the technology for technology's sake approach that schools currently have and brings the focus back to using technology as a tool to improve learning. CLIx is trying to demonstrate a model of in-service education which sees TPD (Teacher Professional Development) on a continuum wherein teachers begin by developing their digital skills but go on to complete certificate courses in chosen subjects to enhance their pedagogical content knowledge. There are three sites for teacher learning in CLIx - the face to face workshops, the community of practice supported via a social media app, Telegram, and a learning platform, TISSx, which is modelled on the lines of Open edX platform (an OER). To increase accessibility, the course is also made available on smartphones.

The methodology adopted for this report is of collaborative action research in which teacher and the researchers are collaborating to develop students' understanding. For the purpose of this paper, teacher's report of one lesson has been used in which she described how she used the tasks The teacher Amarjyoti

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Sinha is a 34 year female secondary mathematics teacher employed in Government secondary school teacher for last 10 years and has the qualification of M.Sc Maths. She was a participant in the CLIx initiative since August 2017. Since then, she has attended two 3 day workshops and one two day workshops based on blended modules (having both hands-on and ICT based activities) of Geometric Reasoning and Proportional Reasoning developed by CLIx. The workshops focused on developing teachers' content knowledge as well as making teachers familiar with the modules by supporting them to go through it as a learner and then making them reflect on how they can support student learning using these modules. The teacher had implemented one of the module in year 2017-2018 and had completed a 4 credit online course on "Reflective Mathematics Teaching" which has 5 units making teachers reflect on using ICT resources for teaching mathematics through engaging deeply with one of the module through study and classroom implementation and then reflecting on students' thinking and learning from mistakes, strategies for assessment of learning and for learning, the affordances and limitations of the different types of resources used in teaching and engaging in mathematical processes to understand the nature of math. She also actively engaged in the mobile chat group discussions in which around 30 math teachers from Chhattisgarh and faculty and team members of CLIx were participant. The purpose of forming this group was to support the development of the communities so that shared understanding and language for teaching of mathematics can be developed. A regular feature in these mobile chat groups was that a puzzle/question/task was posted every week on friday and teachers used to respond. Apart from this teachers used to post their own questions as well as share the photographs and description of the implementation of CLIx modules in their schools with students along with students' work. Due to active participation in the mobile chat groups and implementation of the module in the previous year, Amarjyoti was selected to be mentored for developing her capacity to guide other teachers to use modules in their schools.

The teaching of fraction naming and comparison

The teacher, Amarjyoti Sinha, came across a problem shared in mobile based chat groups which serves as a community for teachers and facilitators to exchange and share new ideas, questions and discuss student responses. The problem was

> Anshul and Gulshan have 3 and 5 rotis. When they were just sitting to have a lunch, a man came who was very hungry. They decided to share the rotis equally among themselves. The man gave 8 coins as a gift for sharing their food. Now the friends had a disagreement about how the coins shoud be distributed amongst them. Anshul felt that the coins should be divided equally but Gulshan felt that he deserved more. They went to a judge who said that Anshul should get 1 coin while Gulshan should get 7 coins. Do you think justice was done? Why or why not?

The teacher engaged in discussion and shared her solution on the group. She got to see solutions from other group members which used ideas different from hers. Most teachers discuss the problems in the groups but there have been very few instances where teachers have used the questions posted in the groups to engage with the students in the classroom. Amarjyoti used the same problem in the classroom to discuss with the ninth grade students. She did so as she was curious to know students' responses to this problem and would also give her opportunity to assess students' understanding. While discussing the problem she realised that students are facing problems in being able to depict the share of one person in the problem. This is an important step of identifying the conceptual gap while teaching as the teachers' next action in the lesson is determined by what she identifies as the gap. She had used the proportional reasoning module developed by CLIx with her students in the computer lab. The unit 1 of the module addresses the concept of fractions by engaging students in problem solving by sharing food in form of cakes or parathas. Students divide the virtual cakes into equal pieces using a cutting tool in the activity and distribute the shares equally to all the members in a group. One of the problem was to divide 5 cakes among 4 students for which the student had arrived at the answer 5/4. At this point in the lesson, when students were having difficulty in depicting one person's share- she asked students to recall what they had done in the problem of 5 cakes shared among 4 people. This is a point in the lesson when the teacher is not only trying to elicit the knowledge that they have acquired through solving other problems in the module but also is trying to make connections by recalling the way they had engaged with the 'sharing' situation earlier where share was more than one whole. However, she had the sensitivity that not all students would have understood why the share of one person is 5/4, and therefore, she decided to ask students if they think answer is correct or whether it should be ⅘. She observed that many of the students seem confused and were not confident that answer is 5/4. She then engaged the class in discussing the situation of comparing ¾ and ⅗ as she was aware from the discussion in the course designed by CLIx for teachers that students often find this confusing since they look at denominator and consider the bigger denominator depicting the bigger fraction. These two problems of the share 5/4 and comparison of ¾ and ⅗ were taken up by the teacher to develop the prerequisite concept of fraction naming and comparison and to solve the problem which was posed.

After discussing the above tasks with students, they were able to understand what 5/4 means and ¾ as the bigger number. Then she supported students in developing the solution of the posed problem. Students were able to come up with solutions wherein 8 rotis were divided among three people the share of each person would be 8/3. They then subtracted 8/3 from 3 to find the share that Anshul gave to the man and 8/3 from 5 to find the share that Gulshan gave to the man. The students were thus able to find that the man got ⅓ roti from Anshul and 7/3 roti from Gulshan and therefore the distribution of 1 coin to Anshul and 7 to Gulshan would be fair.

Discussion

The teacher felt that the discussion of the problem afforded to make the mathematics relevant to daily life for the students. Engaging with digital modules created interest among the students which helped them in understanding the basic ideas necessary for understanding fractions. She found that students who used to consider maths as a difficult subject were also engaging with the module with interest. But she was aware that only engaging with the module in the lab and getting the correct answer of the problem is not necessarily an indication of understanding, therefore she took discussion of those problems again in the classroom and connected with other concepts that they were learning. She felt that as a teacher engaging with ICT resources like the proportional reasoning module, the engagement in the discussion on mobile based chats in communities and engaging in the online course for teachers not only made here aware of the different ideas for teaching mathematics but also allowed opportunities to learn from other teachers and their experiences.

Experiments with Tactile learning

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Abstract

When students think of mathematics and learning mathematics, words such as celebration, joy, beauty, etc are seldom associated with it. To quote Nathalie Sinclair, “A British study found that contemporary lower-secondary students find their mathematics classes to be “TIRED,” that is, characterized by Tedium, Isolation, Rote Learning, Elitism, and Depersonalization”. In the Indian context also, this is quite true, both at the school and college level. One of the questions that teachers ask is “how can we make mathematics learning relevant and, in a manner, that takes into account different learning styles such as visual, auditory and tactile”. In this paper we discuss our experiences where we used tactile methods in mathematics courses.

Introduction

According to Carolyn Yackel, “Tactile mathematics is defined as recognizing deep mathematical concepts through engagement with physical objects”. Having eagerly followed the work of Diana Taimina, Sarah Marie Belcastro, Carolyn Yackel and others, I wanted to see how college students might respond to tactile ways of learning mathematics. Diana Taimina talks about how non- Euclidean geometry was difficult to understand till she found crochet to visualise hyperbolic surfaces. She says “Lobachevsky called his geometry imaginary because it was so greatly in contrast with common sense: so, it remained for me also imaginary until I could experience straightness in the hyperbolic plane in a tactile way”.

There are several areas of mathematics which are amenable to tactile learning. For example, in topology a torus is made starting from a square and identifying the opposite sides. This abstract exercise can be physically realised by making a bead crochet bracelet. The bead crochet artist converts a two-dimensional design with coloured beads to create a bracelet which has multicolour design, that is a tiling of the infinite plane is converted to finite bracelet design. This can be used to ask questions such as creation of torus knots on the bracelet, representation of wallpaper groups or exploring the number of colours needs to colour a map on the torus. Knitting and crocheting can be used to understand the concepts of curvature on positively and negatively curved surfaces. The idea of exponential growth of stiches on a negatively curved surface through crochet was first discovered by Diana Taimina. She used them in her geometry class in Cornell university to physically represent a negatively curved surface where students could actually “see” the lines by folding the surface. Carolyn Yackel uses knitting to explore questions in topology and geometry. Even advanced topics such as Seifert surfaces can be understood through knitting.

Our goal is to introduce and integrate tactile and visual mathematics in undergraduate education. We wanted to understand whether students would react positively to tactile and visual learning in mathematics and in what ways would they respond, what are the ways in which tactile learning could be incorporated in the program?

Our initial hypothesis was that these would be greeted by a mixture of excitement and resistance but then would ultimately be accepted by the student community at large. The reason for this hypothesis was twofold. Firstly, there are students who have different learning styles and some of the tactile/visual learners would be excited. Secondly, this is a liberal education setting and students are exposed to a variety of disciplines and pedagogies which makes them more open to new perspectives. We expected a little bit of resistance due to the general student perception that mathematics is abstract, boring and a notion that any applications or interaction of mathematics has to be in engineering or sciences. In this paper we discuss two such experiments. The first one was a unit in a quantitative reasoning course, called symmetry. The second one was Pi day celebration where second year students engaged with a range of concepts which had strong experimental, tactile component.

Section 1: Experiment one – Symmetry (Quantitative Reasoning Course)

Quantitative reasoning is a compulsory course in the common curriculum of the undergraduate course (BA/BSc) at Azim Premji University. This course runs in different sections and is compulsory for all students. All the Quantitative reasoning courses have the same objective but use different content and pedagogy. In 2015 July (first semester of the first undergraduate batch), four different courses were offered and each course covered different topics. Some of the themes were basics of graph theory using Python, probability, statistics, Game theory, elementary combinatorics, symmetry and calculus.

Our QR course was called "Game theory, counting and symmetry" was taught by Proteep Mallik and Shantha Bhushan and was based on three units with different approaches to quantitative reasoning. i.e., Game theory, elementary combinatorics and symmetry. Specifically, the Learning Outcome were as follows:

- To appreciate symmetry in nature and understand role of geometry in representing it.
- To understand concepts of rotation, translation, reflection in two dimensions, three dimensions.
- To understand tessellation in two dimensions and appreciate the classification of tessellations.
- Through the creation of an art object, understand principles of symmetry

In the beginning of the course we explored a range of patterns such as repetition of motifs, spiral or logarithmic growth pattern, self -similarity, patterns of cracks, spots, crystals. We concentrated on frieze patterns and actually made all seven frieze patterns. Later we used examples from Alhambra and other monuments to motivate these ideas as well. The students actually made patterns on a strip of paper and were able to visualise and actually play around with these frieze patterns. In on the tutorial sessions we even asked them to form mobius strips from these rectangular decorated strips! The drawback of this effort was that we did not introduce co-ordinates as there were a large number of students who had not done mathematics at the higher secondary level.

Then the class explored properties of rotation, translation and reflection in two dimensions and symmetries of platonic solids - tetrahedron, cube, octahedron, dodecahedron, and icosahedron. This was done by using cut-outs which has to stuck together to form these solids.

We then ended this course with understanding tessellation in general and the work of the celebrated artist M.C Escher. The class was quite thrilled to watch a movie clip about Escher and able to see the link between mathematics and art.

For their project, all students made paintings which were tessellations. This involved understanding symmetry and also calculations and aesthetics. Many of the students were systematic and followed the method of making a motif, stencilling, copying it on to the chart paper and then making it aesthetically appealing. Some students did take short cuts. This project required students to submit the process or journey of making the art work. They had to submit the rough sketches and a commentary on why they chose the final motif. Many of the reports are very good and provided insight into the creative process. Anagha (a student in this course) reported that, "It was an enlightening activity to do as playing around with the form of shapes is a method that enhances the understanding of symmetry". A very moving note was written by Aishwarya Anand who says "I started off by tracing the stencil shape I chose onto the middle of a piece of paper and trying to figure out how best the shape fit together with itself. It resulted in a somewhat hexagonal pattern that looked amazing, if I do say so myself.… My original plan was to add mini kaleidoscope-like square-based tessellations of decreasing size into the stencil shape, in strips of alternating colour. I did try playing with colours many times within the shape. I really wanted to make the tessellation out of another series of frieze patterns and tessellations; and I wanted to play with size". Through formal written feedback process and on oral interview with the students, we got the impression that this was an enjoyable module which helped in getting rid of the fear of mathematics.

(Pic1- Display of tessellations made by first year students in QR course)

Section 2. Experiment Two – Pi day

Pi day (14 March) was celebrated by the students doing the Analysis course (as part of applied math minor) and they put up a “mela” where they showcased some mathematical ideas. This was a project in communication of mathematical ideas and students were encouraged to choose topics that were *suitable for their learning style* and this was a graded project. Many were tentative and even anxious because this event was open to all students and faculty. There was enthusiastic participation from the student community and it was like a “mela”. There were poster displays, interactive presentations, games and demonstrations. There was a lot of colour because of the various visual elements such as paintings and craft items. There was a diversity of topics and approaches but there were also fascinating connections between topics.

There were ten student projects on display with themes ranging from symmetry, fractal, Pascals triangle, pattern in music, pattern in the number pi, negatively curved surfaces, music with no pattern etc. This exhibition has a very clear element appealing to the visual, auditory and tactile. The list of Visual elements: Charts, crochet Sierpinski triangle, crochet hyperbolic surface, painting of fractals created by double pendulum. Paintings of tessellations. The tactile elements: Crochet piece of Sierpinski triangle, Crochet hyperbolic surface, Crochet flat disc, Tiles (Fibonacci sequence), wooden pieces (Tower of Hanoi). Paper folding to understand Pythagoras theorem.

Subsection 2.1

Here we describe two interesting projects – Hyperbolic geometry and Fractals.

<u>Hyperbolic Surface</u>: Amrita introduced non- Euclidean geometry specifically Hyperbolic geometry. She used software which helped audience visualise straight lines in positively and negatively curved surfaces. Crochet model were also used to explain curvature of flat and negatively curved surfaces. The idea of shortest lines or geodesics were explored. The fifth postulate of Euclidean geometry-, through a given point outside a fixed a line, there exists exactly one parallel line, was explored. This was an eye opener to many students. It was interested to note that while Amrita herself was more comfortable with abstraction and using software to talk about hyperbolic models, other students like playing with the crochet models and were able to appreciate negatively curved surfaces.

<u>Fractals through Pollocks paintings:</u>

(Pic 2-Double Pendulum fabricated by Krithika Raghavan)

Krithika (economics major, applied mathematics minor) set out to imitate Jackson Pollock in making a painting that was a fractal. She took help from physics faculty and her father to set up the double pendulum. Her account of the project is as follows: “My project for Pi day was inspired by the art of Jackson Pollock. A mathematician named Richard Taylor suspected that Pollock's paintings were fractal in nature and conducted a study on it. Studies found that his paintings were of a fractal dimension of 1.1 to 1.72. His drip paintings were compared to the chaotic system of the double pendulum. So, I tried to test it out for myself. My father and I constructed a double pendulum using wood pieces and made the ratio of the top portion of the pendulum to the bottom portion 1: ¾. I used an ink dripper to absorb paint and attached it firmly to the pendulum. I kept sheets of paper directly underneath the pendulum and a few steps away from it and swung the pendulum several times. While this did not end up with a replication of Pollock's paintings, it did give an idea as to how his paintings might have started out. One of the fascinating ideas I ended up thinking about is how fractals could develop from both order and disorder and the presence of mathematics in art and nature

There is a sense of agency in the student as the student says "I was just experimenting with what can be done. So, I learnt a lot of fractals and a lot about the different ways of painting. Also, this was one of the only times I felt confident when I **was doing** some form of math". This is a very significant thing as the traditional lecture methods give the impression that mathematics is a complete discipline and is "God given" and there is nothing to actually figure out. The freedom to explore, even though limited in terms of time gave the student confidence that she could integrate art, physics, mathematics, economics. She says the" pi day itself was amazing because concepts like chaos and order and it was like everything meeting, all the lines between different subjects were blurry that day"

Subsection 2.2: Outcomes

Science and mathematics faculty were of the opinion that this celebration did demonstrate that students gain when they are given space and time to explore mathematics in ways that suit their learning style. The outcomes can be described through three parameters

1. Learning styles – This project clearly demonstrated that students do have different learning style and that this needs to be accommodated in the curriculum. Some of the projects were predominantly tactile in nature while some had strong visual aspects to it, while one project related to mathematical pattern and aesthetic in music. In conventional courses there is no space and time for this kind of exploration, this model of having project based learning is definitely beneficial for students.
2. Communication: A very clear outcome was that the students enjoyed the mathematical exploration as well as the communication of ideas to a diverse audience. Sreerama V (who worked on Costa arrays) said that "most importantly, by explaining my project over and over again to the flowing audience, I was gradually learning how I can explain the concepts behind my project more simply and elegantly"
3. Mathematics can by fun: Every student (without exception) enjoyed the mathematics of their own project as well as that of others. To quote Sreerama V, "Others' projects were really interesting and creative and the whole exhibition had the ambience of a middle school math project but with much more deeper insights. I was really surprised how with creative approaches; even advanced mathematical ideas can be fun! It motivated me to learn more"
4. Agency: This is perhaps the most enduring in terms of outcomes. Almost all students felt that they were in charge of their learning. Some of the students articulated this quite well. To quote Sreerama (again!) "The major difference is that I was very internally motivated for this project and even though there was a deadline and marks

for it, I enjoyed the freedom to choose a topic and make it creative". Krithika reported that she felt she was actually "doing" mathematics.

5. Aesthetics: Many students and visitors to the Pi day exhibitions talked about the beauty that was either direct or indirect. One student found it "beautiful "that different projects were connected in surprising ways", another found that there was both order and chaos in mathematics and that gave it an element of surprise and beauty. This is consistent with the view of John Dewey and others and expressed very well by, Jonathan M. Borwein in his essay (Aesthetics for the Working Mathematician) " I shall similarly argue for aesthetics before utility. Personally, for my own definition of the aesthetic, I would require (unexpected) simplicity or organization in apparent complexity or chaos" (Nathalie Sinclair 2001). Another student felt that the ability to touch the physical models and explore made mathematics approachable and beautiful.

Section 3. Conclusion

Through these small efforts it is clear that creating such spaces for tactile learning is needed and should be part of the courses. These benefit the student undergoing these learning experiences and the public exhibition/outreach activities help in reaching out to other students who have negative perception of mathematics.

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Proving 'an irrational number as irrational number': An insight into students' responses

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Mathematics is a human activity and has been considered as one of the greatest realms of human intellectual achievement. It is a self-contained mental discipline with its own language, symbolism and organized structure of knowledge. School Mathematics deals with such kind of abstract knowledge mainly at secondary stage. Dealing with the hierarchical structure of number system is one of the examples of that. As a part of mathematics curriculum, different classes of numbers such as natural numbers, whole numbers, integers, rational number followed by Irrational numbers have been introduced in grade IX.

This paper, as a part of teaching exercise of reflecting upon students' written and oral responses, explores X grade students' mathematical thinking about proving an irrational number as 'irrational'. It further discusses how the proving irrationality of a number depends upon the preconceptions that students have about the irrational number in previous grades.

In grade IX, Irrational numbers are introduced as non- rational numbers i.e. numbers which can't be written in the p/q form where p and q are integers and q is not equal to zero. Some of the familiar examples like $\sqrt{2}$, $\sqrt{3}$,$\sqrt{5}$ *etc.* has been provided as a support of the definition. These numbers are also located on the number line. Further, Decimal expansion of rational numbers have been explored by dividing two integers as non- terminating and recurring. From this property of rational numbers, decimal expansion of irrational number is considered as non-recurring and non- repeating. It has also been stated that proof of irrationality will be studied in next grade.

In grade X, proof of irrationality of a number is provided in the textbooks using *contradictory method of proving*. It is interesting to note that one question to prove irrationality is repeated every year in board exams. Therefore, most of the students of grade X are made to learn (memorize) the proof.

Sample and Data resources

The sample of the study constitutes 10^{th} grade Govt. School students of Haryana. Data was collected through two sources: (i) Teachers' own interaction with students (girls) in a mathematics classroom while dealing with proof of irrationality. (ii) Responses of 200 students on the question 'prove that $\sqrt{7}$ is an irrational number' asked in X grade board exam.

Theoretical Considerations

On Psychological and historical ground, the concept of irrational numbers faces two major intuitive obstacles as studied by Fischbein (1995). First is Incommensurable Nature of irrational number i.e. no common unit may be found to measure two different irrational magnitudes. Second is the possibility of having in an interval, two infinite sets of different types of numbers. i.e. set of rational numbers as well as irrational numbers are infinite and can exist in the same interval. further, from epistemological point of view, Zazkis & Sirotic (2004) considered role of representation influencing students' responses with respect of irrationality. Three kinds of representations are considered. The first identifies with the decimal representation of an irrational number. The second representation identifies with the fitting of the irrational numbers on the real number line. The third representation thinks about the connection among incommensurability and the irrational numbers.

Result and discussion

While beginning with proof of irrationality in classroom, number of complexities related to conceptual understanding of an irrational number as well as proving process were noticed.

classroom vignette

T: $\sqrt{2}$ is rational or irrational number?

SS: irrational

Argument s1: because it has root on it.

T: how many support s1?

(except 3-4 students, all hands raised up)

T: what do u say in this regard? (s2)

S2: (she came on the board and wrote a number $\sqrt{4}$) it is not an irrational number because it is a complete square.

T: can we say that sign of square root is not a good indication for judging a number to be rational or irrational?

S2: it is, if there is root sign, we can check (jo sankhya root m likhi hai, kya usse root kat jayega.) (she means a complete square)

S3 (another student who didn't support the first argument): $\sqrt{2}$ is a rational number because it can be expressed in terms of p/q. she said it can be written in terms of $\sqrt{2}$/1

T: who all are agree with S3?

S4: it can be written but we do not know what kind of number $\sqrt{2}$ is and p and q should be integer (aur p/q for me hone ke liye p and q purank hote hai).

T: can we check which kind of number is it?

S4: she started using finding out square root of 2 using division algorithm

T: all of you can try to do this...

S4: claimed that decimal ke baad non-repeating and non-terminating hai. is liye it is irrational number.

T: How can you say this that it is non- repeating and non- terminating? cont…

As it can be observed that students did not feel the need to prove deductively. It may be because of students' notion of considering a number with a root sign as irrational number. One of the reasons could be that students do not consider square root sign as an operation. Rather, it is representation (numeral) for irrational number just like we have for natural number as 1, 2,3 etc. In the further discussion with students, it was observed that students tend to rely on decimal expression to prove its irrationality. On contrary, they rely on finite decimal expression to see the pattern in it. If pattern was not observed till 5 decimal places, it was declared an irrational number. It can also be revealed that students' basic tendency is to have an observable phenomenon to prove. Verification is considered equivalent to proving. In this vignette, teachers' questions however made students think but could not evoke the need to move towards prove irrationality of number deductively.

To further investigate, students' responses related to proof of irrationality were analysed. Some interesting categories were evolved showing students' confusions in conceptual understanding of rational and irrational numbers. It was observed that around 10% of the students could wrote a full proof of theorem '*Prove that* $\sqrt{7}$ *is an irrational number'*. Variety of responses (mathematically not correct) were obtained indicating complexities involved in understanding this theorem. The following categories were evolved:

Category 1: Irrational numbers are not different from other kind of numbers like integers or prime numbers. Example of one of the students' responses: As 7 has no factor other than itself, therefore it is an irrational number. Analysis of the responses reveal that representation of irrational number as $\sqrt{7}$ *may be problematic for student. As mentioned in the above discussion students do not see this root sign as an operation on 7. they consider this sign as a way of writing a number (irrational). Further, as it is argued by Fischbein that concepts of natural, rational, irrational and real numbers are not addressed explicitly and systematically at school level. Rather, focus is merely on technical knowledge, definitions and solving procedures. He referred 'problems raised by the intuitive background without which mathematics is a mere skeleton'. Questions regarding infinitude and incommensurability of irrational number are never addressed at school level.*

2. Irrational number has square root sign on it hence there is no need of a proof.

Almost 40 % of the students gave justification *As* $\sqrt{3}, \sqrt{2}, \sqrt{5}$ *are irrational number therefore* $\sqrt{7}$ $is\ also\ an\ irrational\ number.$ Contrary to the first category where square root sign was meaning less for students, irrationality of a number was recognized with square root sign only. Reason for such kind of responses may be the kind of exposure in previous classes students have for irrational numbers. They have attached the meaning of irrational number with this sign only. It can be concluded that students do not possess understanding of the irrationality. Cognitively, it can be observed that $\sqrt{7}$ is an opaque representation (as termed by Lesh et,al., 1987) i.e. a representation which emphasizes some cognitive aspects and ideas and de-emphasizes others. Therefore, students may identify any number say $\sqrt{4}$ as irrational number.

3. Difficulty in considering contradiction method of proof for proving a statement.

Sample of one of the students' responses

Let us assume that $\sqrt{2}$ is a rational number. so $\sqrt{2}$=p/q where qis not equal to 0. now because $\sqrt{2}$ = p/q hence it is a rational number. so, it is not an irrational number.

Method of contradiction is first time introduced in grade X for proving irrationality. As discussed above that most of the students lack in conceptual understanding of irrational numbers. Assuming an irrational number as rational number give them freedom to think that it may be a rational number as they can see it as $\sqrt{2}/1$. It further leads to the conceptual gap in understanding of Rational numbers i.e. its rigor may not be addressed in school mathematics. Sirotic & Zazkis (2007) explored that gap between formal and intuitive knowledge of rational and irrational numbers. Their findings indicate that underdeveloped intuitions are often related to weakness of formal knowledge and algorithmic experiences.

4. Difficulty in understanding the central idea of proof. i.e. taking p/q as co-prime numbers was another category which was emerged after analysing students' responses. In a proof there is always a central idea on which theorem is built upon. Students in their responses did not mention this idea at all. However, in the proof, contradiction is arrived due to the assumption that p and q are finite.

From the study, it can be concluded that defining irrational numbers as non- rational number is not enough in secondary grades, it leads to lots of misconceptions. Relation between rational and irrational numbers should be emphasized. Questions like richness and density of rational numbers and irrational numbers should be raised in order to evoke students thinking.

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Madhava Activities in West Bengal

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It was an urgent need to have a mathematics competition like ‘’Olympiad for school students’’ in undergraduate (B.Sc) level students as the teaching learning in college mathematics became very dry and bookish and internationally such competitions are there. That dream came true when some enthusiastic mathematicians headed by Prof. S.G. Dani and Prof. V.M.Sholapurkar open up the idea of such a competition named after fourteenth century legendary mathematician Madhava from Kerala, India.

Since 2012 we are organizing Madhava Mathematics Competition(MMC) in West Bengal. It has many positive effects in improving the mathematical culture in colleges of West Bengal. Students, faculties are very happy and they are enjoying its fruits. But as a local coordinator I have experienced many limitations and also observed many scopes of improvements of the activities related to this examination. In Kolkata we also have seven sub centers coordinated by a group of very active faculties from some so called unprivileged colleges in remote areas of West Bengal. Those coordinators also have communicated and shared many important and interesting experiences. In this article we plan to discuss those experiences, limitations and scopes of improvements.

Examination related activities are begins with the form filling or enrollment for the examination usually in the month of October. This is not an online activity but used to be done through department of mathematics of colleges under a center. In one side it is good for students of colleges from remote areas on the other hand teachers feel uncomfortable in dealing with registration fees as liquid money and its documentations and also students from cities are comfortable with online system. It is also some discomfort for center coordinators for not issuing any admit cards or hall tickets. In most of the cases the number of students wrote in the examination is much less than the number of students registered as they forget just because they don’t not gets any admit card. Sending examination materials are very smooth and in every time we have conducted examination smoothly. In every time evaluation related work also went very smoothly. Prize distribution functions were also went very nice but in the year 2012, 2013 prize winners were well distributed to various colleges around Kolkata and that time number of prize winners were also more but from the year 2014 the reputed institutions like Indian

Statistical Institute, IISERs took part and they won most of the prizes. In these years number of prizes is also becoming less.

Though the motto of this competition is not to distribute prizes but to give a chance to the students of colleges to face an examination of true sense, compared to their college/university examinations. We have observed that these days the number of students from ordinary colleges are taking part is becoming less and less as collectively they do not have any chance to win. I think we need to think seriously whether any measures can be taken. Though the syllabus is based on B.Sc second year level in our country containing matrix algebra, calculus, geometry and combinatorics but I think some elementary abstract algebra may also be included. Question pattern is also good but I think we may improve it by including some MCQ question which may have more than one option correct. I think more question related to geometry may be included.
Though very less number of students are prize winners but Madhava Camp gives some oxygen to average students. In every year since 2012 except in 2015 we have conducted 7 to 10 days long Madhava Camp and reputed faculties conducted sessions with students. In each of those Madhava Camps around 50 students participated. It is a very nice idea to mix average with very good. I am witness of molding many averages from very remote colleges to a very good one after attending such camps.

Students from other parts of the country also participated in those camps.
To be inclusive we have initiated 2 or 3 days Madhava Workshops in remote colleges of West Bengal having students with enthusiasm. We have conducted four such and very surprised to see the deep interest of students from the rural part of Bengal to learn mathematics. One of the reasons of this Outreach Programme is that the examination timing of different universities of Bengal are different so it is very difficult to get a common time for all students in Bengal. Even in the national Madhava Camp many gets selected but could not participate because of having their college/university examination.

One of the important lack is that the very less number of girls students are participating in Madhava Mathematics Competition. Keeping this in mind we have open two of our sub centers in Girls Colleges. We have a plan to organize more activities in girls colleges. We have observed that many students join the B.Sc mathematics programme without having even proper interest in mathematics, majority of them used to join by chance not by choice and as a result a large number of failures occur in mathematics B.Sc programme in Indian Universities. I think two possible measures can be taken one of them is arranging some programmes in the history of the development of mathematics as the way we teach some mathematics in classes is not the actual way it came into picture another is we may also have continuous problem solving classes throughout the year so that the fear of mathematics may be removed. I think the learning process must be very joyful. We have a plan to open examination centers in a college of every districts of West Bengal in future and also we would like to establish those centers as learning centers by arranging colloquial lectures by locally reputed mathematicians. In those centers we would plan to organize many activities related to mathematics and its cultures. We believe if we can bring mathematics in our culture and involve every child in mathematics then it would change the intellectual frame of our country. Though the competition is primarily meant for students of Mathematics and Computer but I think it should be open for students in Statistics also. From 2012 onwards almost half of the prize winners and participants of Madhava Camps are from West Bengal.

In conclusion, the Madhava activities in Bengal is very successful and promising. It is very good to have a separate website of MMC having all the informations including previous year questions. I think it has a very good impact on students, they are thinking beyond book by observing these questions. I think it has a very good impact on students, they are thinking beyond book by observing these questions. It is my immense pleasure to thank our chief coordinator Prof.V.M.holapurkar for guiding me in all respect and to congratulate the mathematics loving students of West Bengal.

Teaching and learning Math through Math Manipulative

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Key Words: conceptual Knowledge, Procedural knowledge, Manipulative, Place Value

Mathematics class always challenges to explain the concept to children because in mathematics it is difficult to show or see the math. With help of manipulative we explain. Manipulatives helps to teach mathematics is an innovative method which creative interest among student in classroom. It helps to teacher and student, to teacher, it helps to explain the concept and construct the knowledge among student. To student, it helps to understand and build their knowledge for abstract mathematical concept / ideas, later they could explain the process with conceptual understanding.

According to Piaget experiment with children on math, that shows that children are very active and engaged with math manipulatives in learning process of mathematics. Further, his studies gives understanding us that children they knew number one to three before they come to school i.e. is called subitizing.

Manipulative helps to connect conceptual knowledge and procedural knowledge. Often teachers do not connect both these knowledge for student in learning process. Completing a sum and arriving at an answer alone doesn't mean that the child is good in math.

Teachers should address the errors and misconceptions and often should change the teaching and learning process based on student misconception and errors in each concept. So, in this paper we as authors share our experiences with children that how math manipulative helps to children to gain interest and also to know both conceptual and procedural knowledge in place value.

Based on our own experience and the notion we got from our friends and colleagues regarding the hard spots in teaching math we found that 'the mistakes done by the children while performing four fundamental operations (addition, subtraction, multiplication and division)' is a major problem. The problem begins when they start learning regrouping "Oh, where to put the carry over". For beginners those who are learning the regrouping in addition are surely struggling with this. In most of the regrouping sums the carryover will be ''1' only. So, the children are generalizing that the carryover will always be ‘one’. It is getting worse in the case of subtraction. Without knowing what they are actually doing the children are borrowing the number from the next place. This is the scenario for decades. Even many of us understood about regrouping when we were in our colleges. While discussing about this problems many teachers were saying that the children will never get this understanding of regrouping at the primary level. We started thinking ‘if regrouping is not up to the cognitive level of the children, then why it has been placed in the primary curriculum for years?”. So we decided to make our children to learn this concept of regrouping at the primary level itself.

What is this regrouping means? It is nothing but place value. This paper addresses the way of how a set of children at Savarayalu Nayagar govt girls Primary school are learning the concept of place value from class one(2016) to class three (2018).

This concept of place value is introduced to kids as soon as they start learning number 10. For introducing numbers we were using different materials. Particularly to build on the numbers up to 100 we choose ice sticks. When the children were introduced to number 10 and moves on to the ‘teen numbers’ the ‘bundle and loose’ strategy was used. Students will count the sticks. When they reach ten, immediately they will make it as a bundle. Then again they start counting as ‘one ten and a one’. When the ones adds up to ten again they will make a bundle and continue the counting as ‘two tens and a one’. Some of the children asked questions like

Why should we always make a bundle of ten?

Why it is not some other number?

We as an adult know that our Indian number system is a base ten number system. That’s the reason we are making it as bundles of ten. If our number system is a base 12 or base 20 or base 60 then we would have bundled it correspondingly. But the problem is not knowing the answer. The problem is how to make the children understand it. But *Vygotsky who is an advocate of constructivism says “Having children drill knowledge they had already mastered was useless. Equally useless was trying to teach them knowledge they were not ready to understand.”*(Include reference ... the long 336 page research paper).So the children were just asked to make bundles of ten. At first they count it as one ten, two tens, and three tens and so on. Later I introduce the names as ten, twenty etc. so at the end of class one these children were able to identify numbers in terms of tens and ones.

Students learning place value using 'ice sticks'. Grade 1_2016_SNGGPS

This is the first level of learning place value using ice sticks at grade 1. It is more concrete. Since when they say 43 means they visually see forty three ice sticks, despite of whether it is bundle or loose. The next level is the semi concrete. That is a concrete material with certain level of abstraction. Yes the 'base ten blocks' were used. It is more attractive than the ice sticks. So the children are more attached to it.

The children were exploring the material by themselves. Asked the children to find out how many unit cubes equals the rod, and how many rods makes a mat. Initially the terms cubes, rods and mat were used, but later on introduced the terms such as 'ones, tens and hundreds'. Since they were are not learning four digit numbers (as they are in class three while doing this activity) the thousand cubes were not introduced to them. Using this base ten blocks they learnt to represent up to three digit numbers. This is another important stage. Because:

The first thing with that material is visualizing the units. It is very clear that unit cube is smaller than the ten's rod and the ten's rod is smaller than the hundreds mat and the hundreds mat is smaller than the thousands cube. This gives a clear understanding of the smaller units and bigger units. Also able to understand about why our number system is called as a base ten number system. Because ten ones makes one ten, ten tens makes one hundred and ten hundreds makes one thousand... So every unit is ten times bigger or smaller than its next unit .Students arrange the ones unit cube and could see that ten such unit cubes make the ten's rod.. Similarly they understand that ten such rods make a hundred. Ten such hundred makes a thousand cube.

With the help of the base ten blocks they play a game “Rushing to hundred”. It is a two player’s game. The rule is to roll the dice twice and pick up the corresponding number from the ‘BTB kit’. As soon as they collected ten units they have to exchange them with a rod. The players who exchanges their ten rods and get one hundred mat is the winner of the game. Children played this game very curiously. This game sets a good foundation to the concept of regrouping.

This year the same batch of students was in grade III. *With long term of manipulatives in mathematics, educators have found that students make gains in the many general areas(Heddens; Picciotto, 1998; Sebesta and Martin, 2004).Using manipulative increases students, understanding of place value (Phillips, 1989).* So, these children were exposed to more manipulative. Because for a student who doesn’t understand a concept with one TLM, may understand the same concept with another. So, this year we introduced two more tools along with ‘BTB kit’. ‘Monk money’ and ‘Tokens’ are the other tools. These children used the above three tools to do ‘place value Addition and Subtraction’

Addition: For adding two three digit numbers, say for example 326 + 245. They first take 3 hundreds, 2 tens and 6 ones then they take 2 hundreds, 4 tens and 5 ones. Now they altogether make- 5 hundreds, 6 tens and 11 ones. At this stage the children regroup the ones as ‘1 ten and a one’. So now the total becomes 571.

Subtraction: For example we take 326 – 154. Children first take 3 hundreds, 2 tens and 6 ones. Then they have to take away 154 from 326. They can take 4 ones from 6 ones, as they move on to the tens they got stuck up as they couldn’t take away 5 tens from 2 tens. At this stage they do the regrouping. They take one hundred from three hundreds and split it up into 10 tens. Now they have totally 12 tens. It is now possible for them to take away 5 tens from 12 tens. Then they take away 1 hundred from the remaining 2 hundred. The answer is 172.

Children did the same procedure using three different tools. Their conceptual understanding is definitely better than any other students who are learning addition and subtraction in a usual ‘Carryover and borrowing’ method.

At Savarayalu Nayagar School, during the first term a math expo was arranged by us. In which many of the stack holders- Officials from the Education Department, Teachers and Head Teachers from the nearby schools and the parents visited. They highly appreciated the talent and the depth of the mathematical understanding of these children. ‘Place value Addition and Subtraction’ is one among the theme of that expo that attracted the attention of the visitors. These children can do the addition and subtraction with more understanding and less mistakes. This is only possible because of ‘Math manipulative’.







Conclusion: The children in primary classes always want to play. *Nathalie Sinclair a researcher has argued that children have a natural attraction towards mathematics up to grade three. But after fourth grade with about 40% saying that they hate math.* This is because of the way mathematics is taught. When they see manipulative they feel math class is enjoyable. As they are doing math with the help of manipulative they feel as if they are playing in math. While playing they learn mathematics. Manipulative helps teacher to engage students very active in math class. Teacher should plan hands on activities to give exposure and explain the concept of math taking the support of manipulative. Manipulative plays an important role in scaffolding. *As the children starts constructing their own knowledge teacher can slowly withdraw the support* (Bigge &Shermis, 1999). Manipulative helps the students those who need remedial and also for those who are good in math to go ahead further to the next level of the concept. In that aspect Manipulative helps to achieve the learning needs of the whole class.

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Teaching Inquiry based Mathematics- An Anecdotal Case Study

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Introduction

This paper was written basis the experience of teaching Maths in an inquiry based way at Al Qamar Academy, Chennai, to students in grades 4-6 using the Montessori methodology and the Connected Maths Curriculum[1].

Al Qamar Academy is an alternative style micro school in Chennai, India. The school uses Montessori methodology for the Primary classes. For middle school, various curricula are used for different subjects. The school does not give exams or homework.

Inquiry Based Maths Teaching

Inquiry Based Maths is a way of teaching maths which uses real life scenarios and problems as opposed to explicitly teaching math techniques. Students use their own insights, thoughts and understanding to tackle problems, thus deepening their understanding.

Inquiry based Maths Teaching has the following five criteria[2]:

- **Focus on Process:** The process of mathematical thinking is given high priority. Students are encouraged to think for themselves and use different approaches to solve problems.

- **Discussion oriented classes**: Discussion is a key component of the maths classes where students explain their thinking to the teacher and each other. The discussions bring out the variety of approaches that students have used and students get exposed to different kinds of thinking.

- **Investigation/ Inquiry:** Students investigate problems to develop insights and solve, as opposed to being explicitly taught a math technique.

- **Collaborative Learning**: Children are paired together to work on Investigations in similar ability groups.

- **Role of a teacher** as a facilitator not dispenser of knowledge. Teachers provide guidance, a sounding board and facilitate discussions, provide feedback and direction rather than answers. This helps the children construct their understanding using either concrete material, past experiences or their own original thinking. The teacher also uses questions to elicit the student's understanding and identify misconceptions.

Inquiry based Maths teaching Implementation at the school

Montessori

The Montessori Method has mixed age group classes where children within a 3 year band constitute a cohort. There are minimal textbooks, no fixed seating, no fixed timetable. Children choose their work independently. Group work is encouraged. Concepts are taught in individual or small group settings. The curriculum is personalised for each child.

The Montessori Math material is used from the early years to introduce the child to numeracy, decimal system, math operations. The material is used in Grades 4 & 5 to teach concepts of Prime & Composite numbers, LCM, HCF, Fractions, Decimals, Mensuration. The Geometry material helps the child build an understanding of both 2 & 3D shapes and their properties. Children gradually and systematically move from the concrete to the abstract over the year. They develop and share their insights into mathematical concepts while building their fluency. Children are free to use math material repetitively till they feel they have consolidated the learning.

Connected Maths

The Connected Maths Project (CMP) is an American problem solving based curriculum which is taught in an inquiry based way. Mathematical concepts are ingrained in a set of sequenced tasks to be explored in detail leading to students building a conscious/ subconscious understanding.

Each topic begins with guided investigation questions, a game to investigate etc. The students work in groups and come up with their solutions which they can then present to the class on vertical surfaces, charts, the board, etc. Each group explains their thinking process and how they arrived at their results. This gives an excellent opportunity for a myriad discussion on the topic and clarity of thought. It's then followed by questions ranging on the application, connection and extension scale.

Mistakes are positively treated. "A Mistake is a good thing" is a constant refrain because it encourages children to try alternative approaches rather than getting fixated on getting the "right" answer. Mistakes are also a natural by-product in an inquiry based method as children try out different approaches to a problem.

Logic & Math puzzles / Extended Maths Class

These supplement the Montessori/ CMP work and are worked on in a class setting or individually. Puzzles include geometry and shape puzzles, math puzzles and logic.

Caveat: Small Sample size

While Al Qamar has been using the Connected Maths curriculum for the past 5 years, and Montessori method for 10 years, the class sizes have been very small.

Results of the Inquiry based teaching

No formal metrics or observation criteria have been used. The results are anecdotal stories.

- **Deeper understanding**

We had covered the chapter on Prime/ Composite Numbers and Factors in Connected Maths. One of the problems was : Imagine in a school, there are 1000 lockers. On the first day of school, all the students line up. Student 1 runs through and opens all the lockers. Student 2 runs through next and closes Lockers2,4,6,8 and so on. Student 3 runs through and changes the state of (closes the open ones, opens the closed ones) of lockers 3,6,9,12 etc. In the end, after the 1000th kid has run through, which lockers will be open? Why?

They were initially overwhelmed by the fact that they had to go through the motion with 1000 students and 1000 doors! The hint by G.Polya in the textbook to solve a simplified version of the problem and search for patterns helped them. They chose to work with 10-30 doors to seek patterns. And then they were buzzing with multiples - open, close, open.

"The prime numbers will be closed".. came the first observation. Number 1 opens them and they close themselves. They have only two factors. An Aha! moment there. Soon they solved the problem for 10-30 doors, checked the open doors and found 1,4,9,16,25 to be open. "Square numbers", came the answer from different teams. We further explored why square numbered doors were open-"because they have an odd number of factors, they don't have all factors pairs. So, the open door will remain open " came the replies.

- **Cross Connections/ Applying learning in different contexts**

The children had recently been taught Fractions using the Montessori concrete material and they had been using the material extensively. We were reading a story from "The man who counted" by Malba Tahan, where 35 camels are to *be divided as 1/2, 1/3 and 1/9 among a man's heirs. The solution lay in adding an extra camel and after a satisfactory division, 2 camels are left over. Some 5th grade children immediately saw that the fractions didn't add up to 1 and hence there would be remaining camels. Another 4th grade child calculated the number of camels per share based on 35 and saw that it didn't add up to 35 in the first place hence the extra camel. This gave a clear insight into their understanding of the chapter of fractions we had just covered.*

- **Developing Insights/ Math Discoveries**

*The class did investigation on how to find the sum of the interior angles of a polygon. They had two situations. One where it's broken up into triangles through a single vertex. And the other where a point is marked anywhere inside the polygon and then connected to the vertices to break up the polygon into triangles. The question was whether both methods would work. On discussion, the students felt the first method would work and the second would not. One student realized that it's sum of the interior and of the triangle formed minus out the whole angle formed at the centre. They did this with various polygons to arrive at the general formula for the angle sum is (n-2)*180.*

- **Non-Traditional thinking/ Different thinking styles**

*However, one child had a different way of approaching the Locker Room problem (Stated above). She simply wrote down numbers 1-100 and told me "The answer is square numbers". Asking her the reason, she replied "It's because there are no two two "people" (sic), no three three people and so on." I had to ask her to explain. "3*2=6, there is a person 3 and 2 but 2*2=4, 3*3=9, there are no 2 of the same people, so only one will touch the door. So, after the other factors close it that single person will open it". She worked out the idea and got her answer the other way around than done by most students.*

- **Investigative Mindset/ Seeing patterns**

I was presenting the divisibility rules using Montessori material and how to arrive at them to a group of fourth graders and the students were marvelling at the simplicity of it. We did the rule for 2, 3,4,5,6 and then 8. One fourth grader couldn't understand why 7 has no rule. She decided to derive it herself.

*"Every thousand to be divisible by 7, needs a 1 more. Every hundred gives away a 2 to be divisible, i.e. 98 is divisible and 2 is left over for every hundred. So, the hundreds digit*2. Every tens digit leaves a remainder 3. So, it's tens digit*3. And the units are taken 1s (+) 10*3(+) 100*2(+) 1000*1(-). (Notation - + Has. - needs) So every thousand needs a 1, then ten thousands need ten, 8 hundred thousand will need an 800 and so on. This makes it very difficult to test beyond the thousands digit, so the test fails, she concluded.*

*Why do ten thousands need ten they may need only 3...So it becomes: 1s (+) 10*3(+) 100*2(+) 1000*1(-). 10,000*3(-) 100,000*2(-) 1,000,000*1(-) 10,000,000*3(-). And so on in the same pattern.So, all the negatives add up and are subtracted from the positives. If the result is divisible by 7 the number is divisible. If not it's not divisible."*

- **Problem Solving approach**

The weekly Math Puzzle card asked children which numbers cannot be expressed as the sum of 2 or more consecutive numbers. One 5th grade child made a list of numbers and sorted them into "cans" and "cannots". She observed a pattern. She saw that each odd number could be expressed as a sum of consecutive numbers. Then she saw that some of the even numbers - she called them "doubling numbers" 1,2,4,8,16.... cannot be represented as sum of consecutive positive integers while all other even numbers can. She reasoned - "2 is a 'cannot ' number, and the rest are its doubles. So even they can't be sums of consecutive numbers"

- **Persistence with a task**

Then we've been also introducing geometric solid puzzles - a star shape, tessellations and the Tower of Hanoi. Students struggle with the puzzle and persist for days till they have solved it. In the process, they make observations and ask deep questions "What shape will form inside the solid?" "How many faces will it have?"

- **Strategizing approach**

The Connected Maths curriculum has embedded various games to consolidate learning. We had been playing the Factor Game where player one circles a number on a board of 1-30 and player 2 in turn circles it's proper factors and they both score the respective points. Already circled numbers can't be circled again. A player shouldn't circle a number where the opponent is unable to score a single point, if that's not possible the game ends. The aim being to score the highest points.

We played the Factor Game many times as the kids started to realize various strategies to win and wanted to try them out. It was followed by a complete investigation on the best and worst first moves, where they noticed the best first move were numbers which had no factors except 1 and that these were prime numbers and the composite with most factors to be the worst. They could stretch their thinking and strategize for a 100-game board.

In the Tower of Hanoi problem (mentioned above) the children worked out the minimum number of moves required to make towers of various sizes and also which peg they needed to start with in order to make the tower in the last one.

Shortcomings of Inquiry based teaching

- *Lack of precedent* – Inquiry based math teaching is very untried in India. Hence there is no precedent about the results from this approach.
- *Lack of Community* – Again as this is a new and uncharted terrain, it's difficult to get guidance or support.
- *Language intensive* – Given the emphasis on real life contexts, the Connected Maths curriculum is very language intensive. Hence ESL learners struggle to cope.
- *Cultural Context* – The Connected Maths curriculum is American and children to find it hard to directly relate to the word problems.
- *Math fluency* – Children do need to develop math fluency and remember math facts. This method may not lead to developing these.

Conclusion

The results inquiry based teaching has produced for the school has been in terms of children enjoying the subject, confidence to work out their own solutions, ability to explain their thinking process, ability to see patterns.

Most children have expressed their liking for Maths. A large number voluntarily chooses to do maths work in an environment where children are free to choose their work. Children have also gained confidence especially ones for whom remembering number facts is a challenge. They realise that it's the Math thinking that is important – recognizing patterns, developing creative solutions, and not only mastery of technique. Children enjoy spending time with Maths and developing their own unique solutions.

We believe that Inquiry based teaching can reverse some of the negativity and fear associated with maths.

References

1 https://connectedmath.msu.edu/

[2] Inquiry Teaching: Models of Instruction: What is Inquiry Teaching and how do you do it? https://mathteachingstrategies.wordpress.com/2008/11/24/inquiry-based-teaching/

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